Berezin Integrals

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In Feynman's version of quantum mechanics, quantum amplitudes are constructed as integrals over all "classical trajectories" connecting initial and final states weighted by a complex action. This idea works on a formal level even when the particles involved have symmetric statistics (or are Bosons in the terminology of Physics). For particles like electrons or protons which have anti-symmetric statistics (i.e., Fermions) the path integral representation requires an anti symmetric substitute for the Lebesgue integral which is known as the Berezin integral. One "integrates" Grassmann valued functions rather than complex functions. Path integrals in the continuum setting are difficult to make sense of but on a finite lattice the Berezin integral turns out to be a construct of finite dimensional Linear Algebra. For discrete actions of the type that are relevant for gauge theories "integrating out the degrees of freedom for a finite domain" produces a state on the boundary which should be characterized as an appropriate vector in a suitable representation of a Clifford algebra. This has been worked out in the very simplest one dimensional cases but does not seem to have been looked at for higher dimensional lattices. Ultimately, the game is to reformulate the Berezin construction in a way that makes possible a mathematically well defined passage to the continuum but even on finite lattices there are interesting mathematical problems that do not involve anything more than an understanding of linear algebra.