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10/26

Problems: p289 #1

Geometry

Why study Euclidean geometry?

- Profoundly influential on scientific thought/methods for past 2000+ years
- Excellent way to begin learning deductive reasoning
- Beautiful "toy" example of mathematical theory (i.e. proof, axioms, ...)

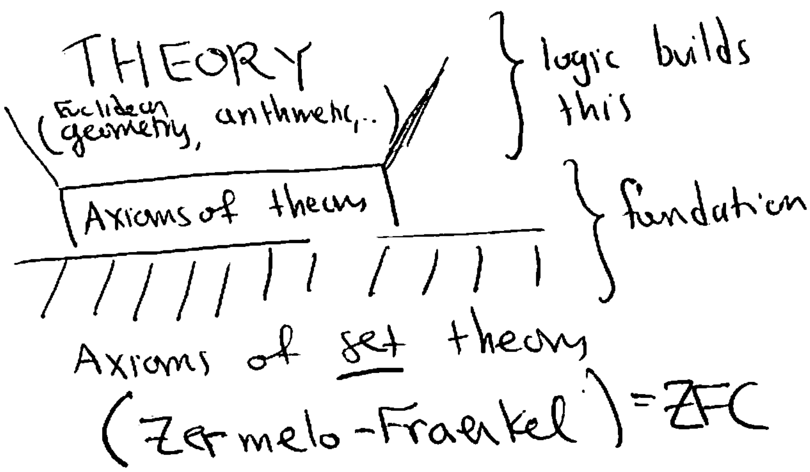
Why is proof important?

- Allows us to be certain about things we may not be able to experimentally or physically verify.
- Often sheds light on why something is true
- Many true statements can seem false, and many false statements can seem true. Only with proof can we differentiate.
- Can help to generalize the statement

$$\boxed{\text{Axioms}} \xrightarrow[\text{deduction}]{\text{logic}} \boxed{\text{theorems}}$$

Axioms = Statements we <u>accept</u> as true,
but which can not themselves be
proven to be true

More specifically,



* The axioms are the "rules of the game"
The theory building is "the game"

→ Many mathematicians / philosophers can / do
debate ~~the~~ axioms, but I don't consider
this activity to be mathematics.

Q: What choice(s) do we have with regard to the axioms?

Def: A set of axioms is <u>complete</u> if any statement is provably true or false from the axioms.

Def: A set of axioms is <u>consistent</u> if there is no statement S which is provably true <u>and</u> false from the axioms.

Th (Gödel) 1930: No (reasonable) set of axioms is both consistent and complete. In particular, consistent axioms $\Longrightarrow$ $\exists$ statements which can't be proved true or false.

"Example": This sentence is false.

Ex: $R = \{ S \mid S \text{ is a set and } S \notin S \}$

Then $R \in R \Longrightarrow R \notin R$ so $R \in R \Longleftrightarrow R \notin R$.

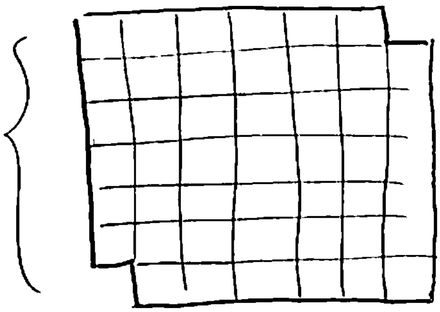
$R \notin R \Longrightarrow R \in R$

<u>problems</u>: Need better axioms of set theory which rule <u>out</u> R being a set.

Examples of the power of proof

Consider 8x8 grid with opposite corners removed

62 squares



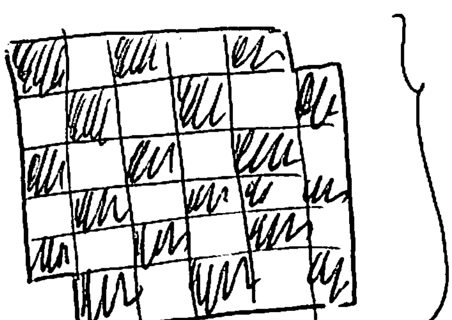
 = 1x2 block (Domino)

Q: Can the grid be "covered" tiled with 31 Dominoes?

Try as you might, you won't find a tiling that does the job.

* We can't just try <u>all</u> possible tilings, as there are <u>too</u> many!

Instead, we <u>prove</u> that no tiling will work.



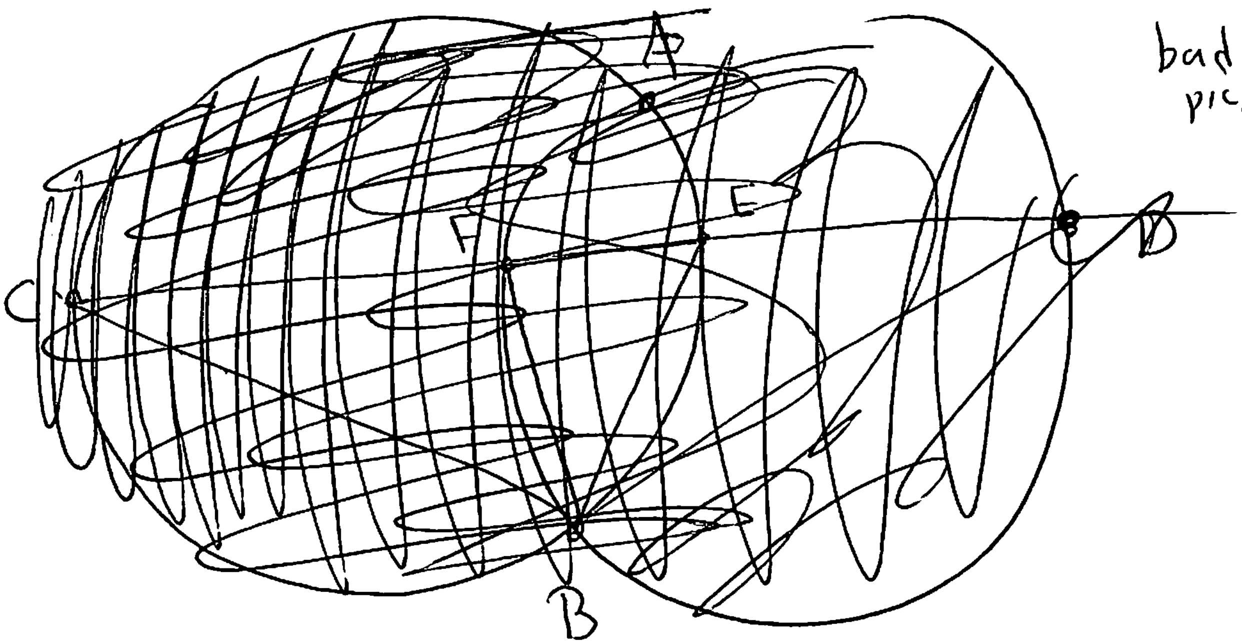
color as checkerboard

As opp corners have <u>same</u> color, there are 32 black & 30 white squares. Any domino covers <u>one</u> white and <u>one</u> black square, so 31 dominoes can't cover 32 black & 30 white squares.

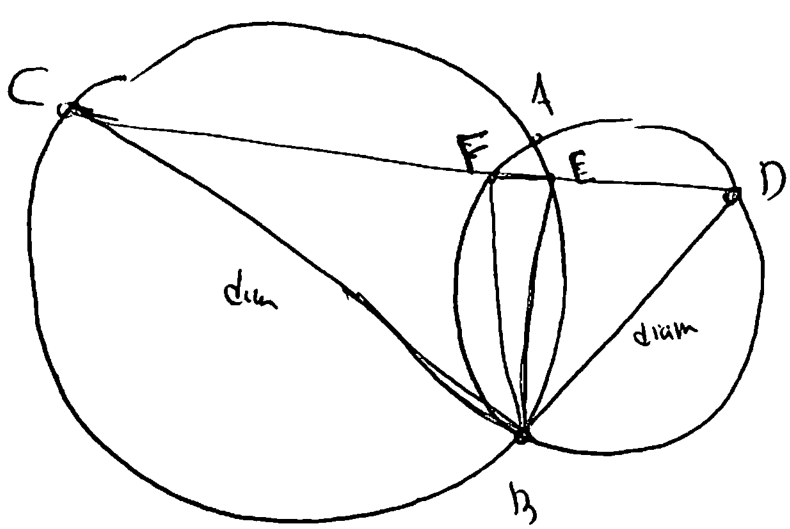
Must be careful in proof to not make unwarranted assumptions (Clearly identify all axioms being used).

Ex: "Thm": There exists a triangle in the plane with two right angles.

"proof":



bad picture

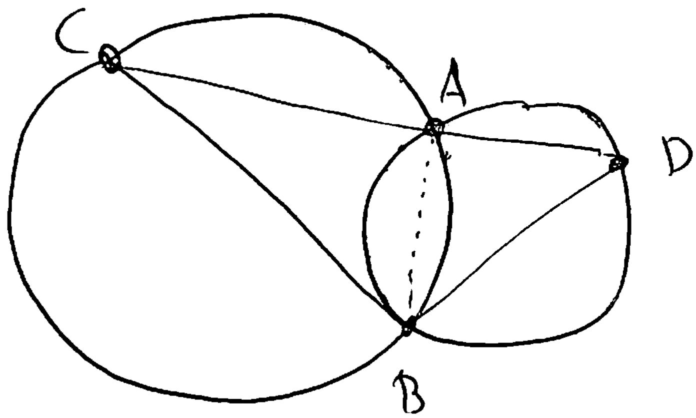


$m\angle BEC = 90^\circ$

$m\angle BFD = 90^\circ$

Conclude.

Of course, the picture makes the false assumption that $\overline{CD}$ intersects the two circles at different points. In fact, $\overline{CD}$ intersects both circles at A.



Moral: Must be careful about assumptions, and especially wary of pictures.