10-5
407 Real functions

<u>Def</u>: A <u>real</u> <u>function</u> is any function f whose domain and range are both subsets of $\mathbb{R}$.

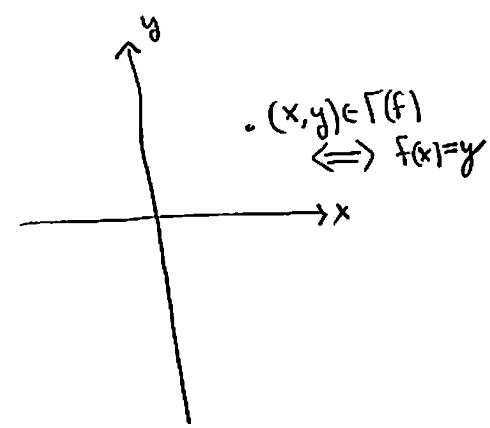
* Such functions are the emphasis of HS. pre-calc, calc.

<u>Q</u>: Why do we focus on real functions?

⋮

<u>A</u>: Because we can graph them in the cartesian plane!

$$\begin{array}{ccc} \mathbb{R} & & \mathbb{R} \\ \cup\!| & & \cup\!| \\ f: D & \longrightarrow & f(D) \\ \| & & \| \\ \text{domain} & & \text{range} \\ x = \text{dep. var} & & y = \text{indep var} \end{array} \longrightarrow$$



Typically, HS math focusses on a small number of "categories" of real functions.

These are:

1. Polynomial fns:

$$y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0, \quad a_i \in \mathbb{R}$$

Domain: $\mathbb{R}$

2. Rational functions

$$y = \frac{p(x)}{q(x)}, \quad p, q \text{ polynomial}$$

Domain $\{x \in \mathbb{R} \mid q(x) \neq 0\}$

3. Exponential functions

$$y = b^x, \quad b \in \mathbb{R}, \quad b \neq 1$$

Domain $\mathbb{R}$

4. Logs

$$y = \log_b(x), \quad b \in \mathbb{R}, \; b \neq 1$$

Domain: $\mathbb{R}_{>0}$

5. Trig fns:

$$y = \sin, \cos, \tan, \cot, \underset{}{\overset{\frac{1}{\cos} \,=}{\sec}}, \overset{\frac{1}{\sin} \,=}{\csc}$$

domains: $\mathbb{R}$, $\mathbb{R}$, $\mathbb{R} - \{\frac{2n+1}{2}\pi : n \in \mathbb{Z}\}$, $\mathbb{R} - \{n\pi : n \in \mathbb{Z}\}$

6. Inverse trig fns ....

7. Sequences / series $\{a_n\}$, Domain $= \mathbb{N}$, or subset of $\mathbb{Z}$.

Analyzing real functions

In order to understend the properties / behavior of real functions we analyze

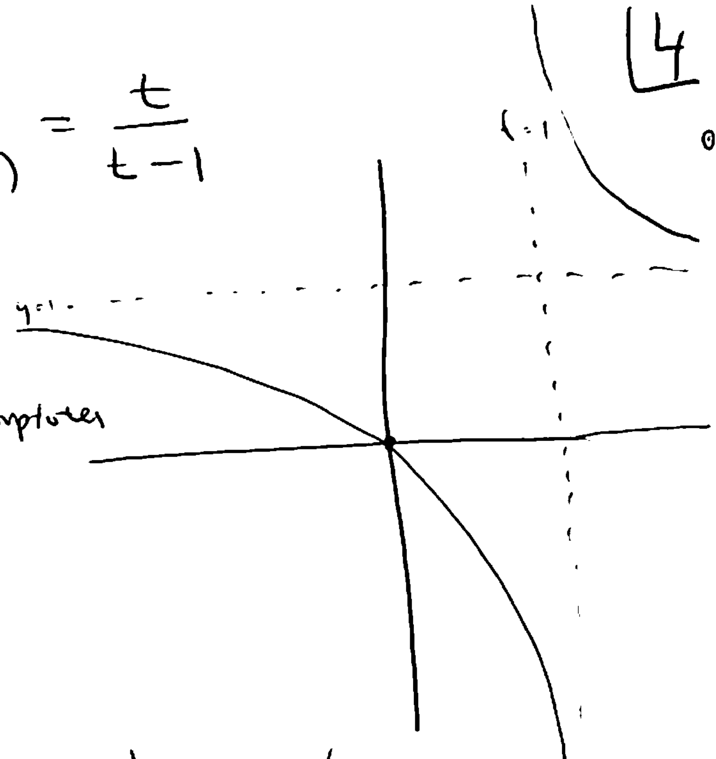
1. Domain.
2. Singularities / asymptotes
3. Range
4. Zeroes
5. Critical points: (relative) max/min
6. Increasing / decreasing concavity, inflection
7. "extremal" behavior ($x \to \pm\infty$, $x \to$ singularity)
8. General props
   - cts
   - diff'ble
   - integrable
   - taylor series
9. Special props ~~test~~
   - is f a composite?
   - even/odd function?
   - Symmetry?
   - periodicity?
10. Applications
    - where does f occur in "real life"?

<u>Ex</u>: $f(x) = mx + b, \quad m > 0.$

1. Domain: $x \in \mathbb{R}$
2. <u>No</u> sings/asympt.
3. Range: $y \in \mathbb{R}$
4. Single zero @ $x = -b/m$
5. No max/min
6. Increasing, concavity $= 0$, constant derivative
7. $\lim_{x \to \infty} f(x) = \infty$, $\lim_{x \to -\infty} f(x) = -\infty$
8. cts ✓, $f' = m$, integrable, $\int f\,dx = \frac{mx^2}{2} + bx + c$, taylor $= b + mx$.
9. <u>Composite</u> of Ex $h(x) = x + b$, $g(x) = mx$ $\quad f = h \circ g$

   <u>odd</u> iff $b = 0$
10. Lin fns used to model <u>constant</u> change, approximate more complicated fns.

Ex: $f(t) = \dfrac{t^2 - t}{t^2 - 3t + 2} = \dfrac{t(t-1)}{(t-1)(t-2)} = \dfrac{t}{t-1}$



1. Domain: $\mathbb{R} - \{1, 2\}$
2. Singularities: $t = 1$, $t = 2$ ← asymptotes
   ↑ removable
3. Range:
4. Zeros: $t = 0$
5. Critical pts: $f'(t) = \dfrac{(t-1)\cdot 1 - t}{(t-1)^2} = \dfrac{-1}{(t-1)^2} \neq 0.$

   no critical pts,
   no max/min

6. $f'(t) < 0$ for all $t \neq 1$, so f is decreasing everywhere ~~it~~

~~8~~. $f''(t) = \dfrac{2}{(t-1)^3}$ > 0 if $t > 1$ cc up; < 0 if $t < 1$ cc down. no zeros.

f'' has no zeros → no points of infl.

~~10~~.

7. $\lim_{x \to 2} f(x) = 2$ $\quad \lim_{x \to +\infty} f(x) = 1$

$\lim_{x \to 1^+} f(x) = +\infty$ $\quad \lim_{x \to -\infty} f(x) = 1$

$\lim_{x \to 1^-} f(x) = -\infty$

8. - Cts, diff except @ $t = 1, 2$
   - Integrable
   - $f(t) = \dfrac{1}{1 - \frac{1}{t}} = 1 + \frac{1}{t} + \frac{1}{t^2} \ldots$ $t > 1$

   $= \dfrac{-t}{1-t} = -t - t^2 - t^3 \ldots$ $t < 1$

9. Composite of $h(t) = \frac{1}{t}$

   $g(t) = 1 - \frac{1}{t}$

   $h \circ g = f$

10. Rat'l function -