Problems: p. 320 # 1, 4

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12-5 (M): (5): Greg, Raquel, Caty, Lynette, Dustin

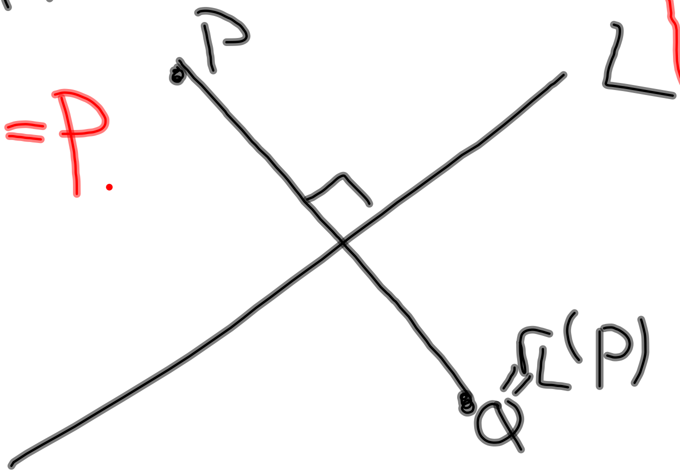
12-7 (W): (4) Michael, Cameron, Levi, Jacinta

Reflections:

L = line in $\mathbb{R}^2$

$r_L(P)$ = reflection of P across L

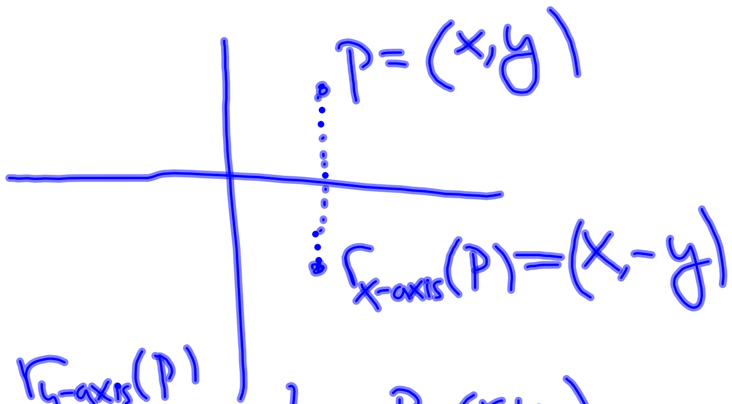
$r_L(P) = P.$



$\overline{QP}$ has L as its $\perp$ bisector

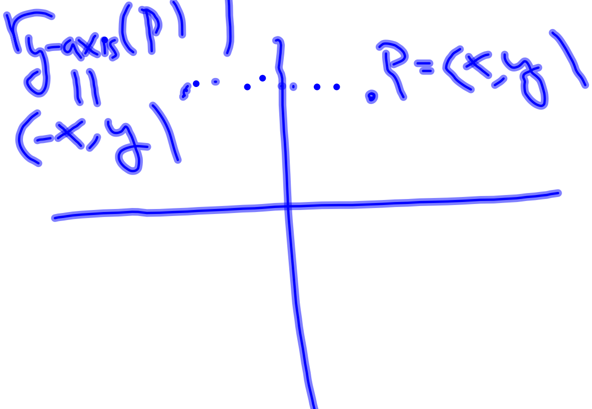
3 special cases

1) $L = x\text{-axis}$



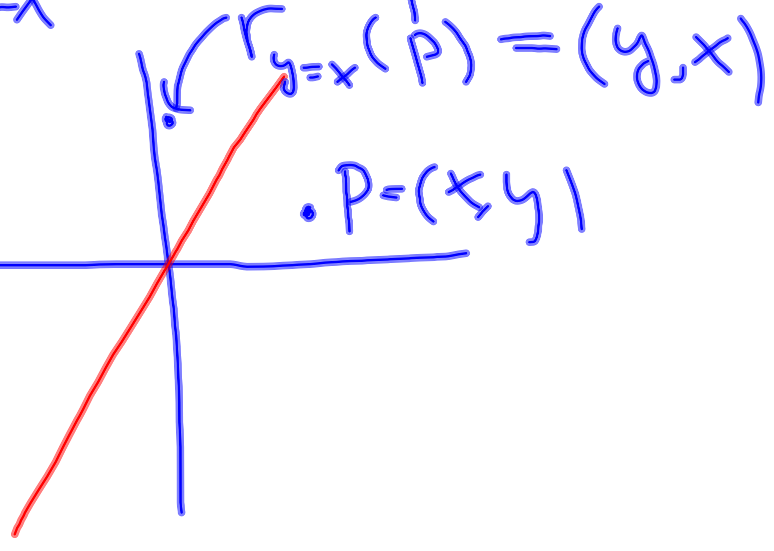
$$[r_{x\text{-axis}}] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

2) $L = y\text{-axis}$



$$[r_{y\text{-axis}}] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

3) $L = \text{line } y=x$



$$[r_{y=x}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Let's figure out a formula for

r_{L_ϕ} = reflection across L_ϕ

Trick: Rotate coordinate axes!

$$R_{-\phi} \circ r_{L_\phi} = r_x \circ R_{-\phi}$$

so $\underbrace{R_\phi \circ R_{-\phi} \circ r_{L_\phi}}_{r_{L_\phi} \,=} = R_\phi \circ r_x \circ R_{-\phi}$

Monday: $[R_\phi] = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$

$$[r_{L_\phi}] = [R_\phi \circ r_x \circ R_{-\phi}] = [R_\phi] \cdot [r_x] \cdot [R_{-\phi}]$$

$$= \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{bmatrix}$$

$$\begin{bmatrix} \cos^2\phi - \sin^2\phi & 2\cos\phi\sin\phi \\ 2\cos\phi\sin\phi & -(\cos^2\phi - \sin^2\phi) \end{bmatrix} = \begin{bmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{bmatrix}$$

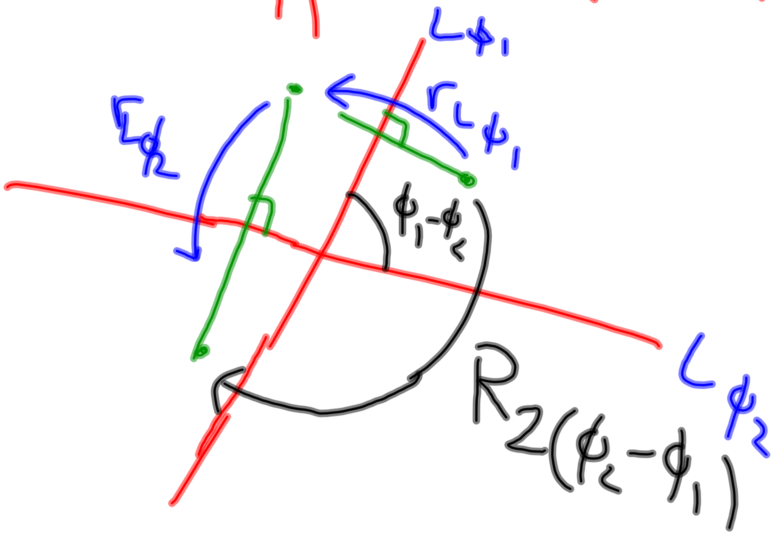
$$= \begin{bmatrix} \cos(2\phi) & -\sin(2\phi) \\ \sin(2\phi) & \cos(2\phi) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = [R_{2\phi}][r_x]$$

In particular:

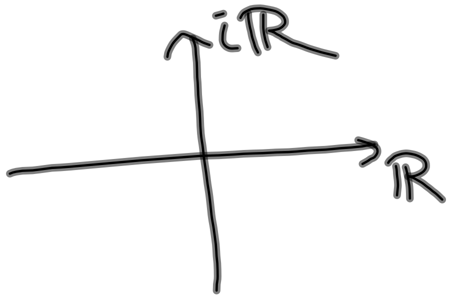
1) $r_{L_\phi}(x,y) = (x\cos 2\phi + y\sin 2\phi,\ x\sin 2\phi - y\cos 2\phi)$

2) $r_{L_\phi} \circ r_x = R_{2\phi} \circ r_x \circ r_x = R_{2\phi}$

<u>Corollary</u>: $r_{L_{\phi_2}} \circ r_{L_{\phi_1}} = R_{2(\phi_2 - \phi_1)}$



Complex picture



Complex conjugation

$$r_{\mathbb{R}}(z) := \bar{z} = a - bi, \text{ where } z = a + bi$$

$$\updownarrow$$

$$r_{x\,\text{axis}}$$

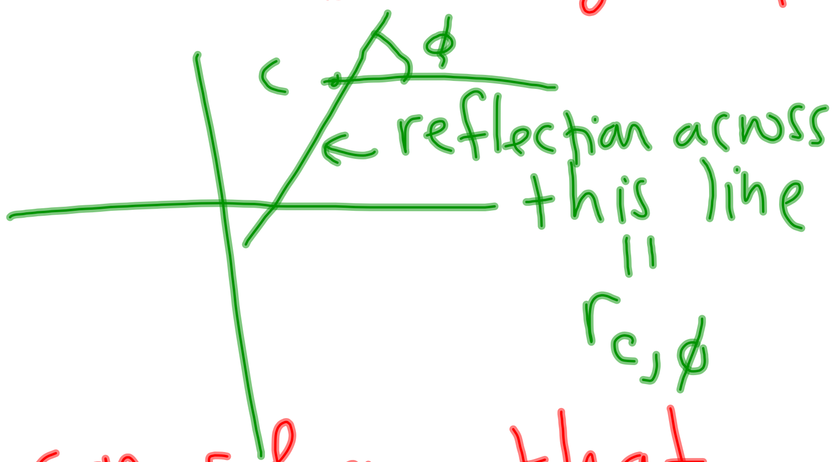
Last time: R_ϕ corresponds to mult. by $z_\phi := \cos\phi + i\sin\phi = e^{i\phi}$

<u>So</u>: $r_{L_\phi} = R_{2\phi} \circ r_x$, i.e. $r_{L_\phi}(z) = z_{2\phi} \cdot \bar{z}$

For a general line (maybe not containing origin)

$$r_{c,\phi} = z_{2\phi} \cdot \overline{(z-c)} + c$$

where $r_{c,\phi}$ = reflection across line through c that forms an angle of ϕ w/ x-axis.



Using this, can show that

$$r_{c_2,\phi} \circ r_{c_1,\phi} = z + \{\text{something}\}$$
$$= \underline{\text{translation}}$$