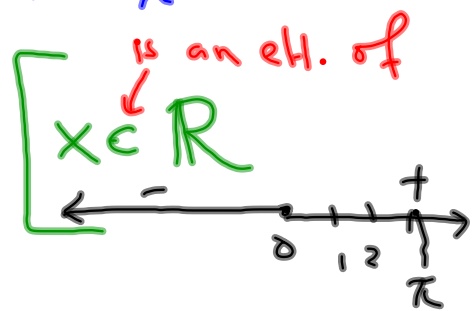


* Definitions are crucial!

Ex: For a real number x
how to define $|x|$?



1) $|x|$ = the dist on # line
from x to 0 .

$$2) |x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases}$$

William says:

Fact: These defs
are log. equiv.

$$3) |x| = \sqrt{x^2}$$

Some defs are better than others
(in certain situations).

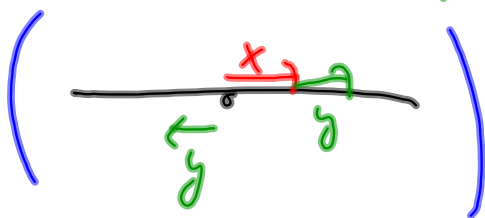
Key props of abs val

if and only if
 \iff
 Defs
 2 (Kim)
 1 (Levi)

a) $|x| \geq 0$ and $|x|=0$ iff $x=0$.

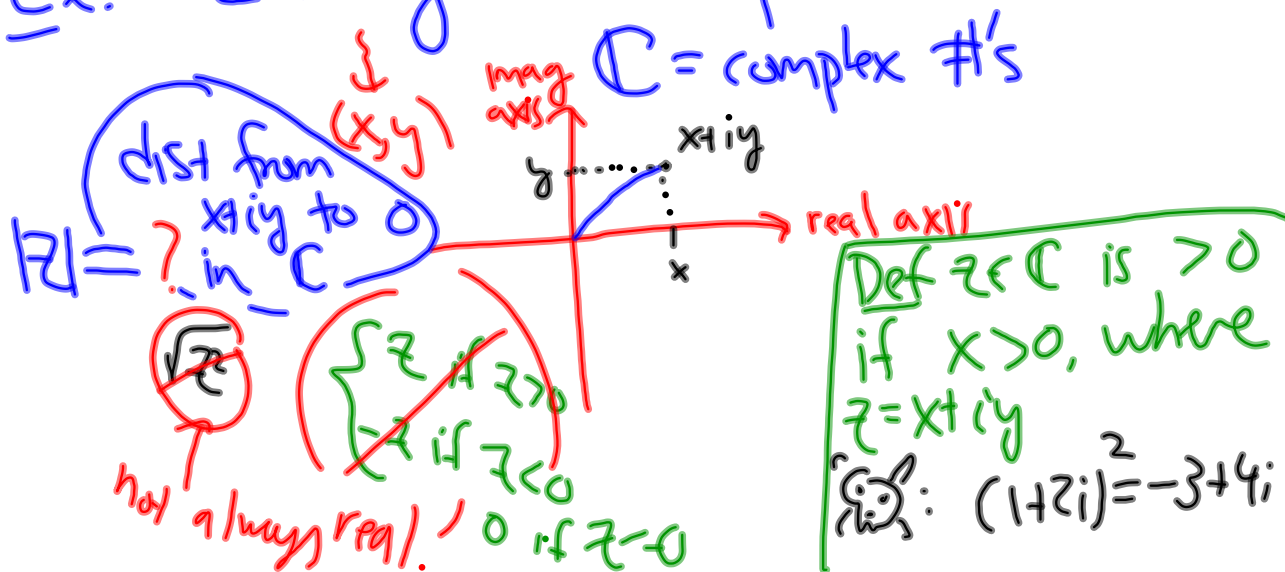
b) $|xy| = |x| \cdot |y|$ ← Def 3

c) $|x+y| \leq |x| + |y|$ ← Def 1.



* The most important thing in making def's: Ability to generalize.

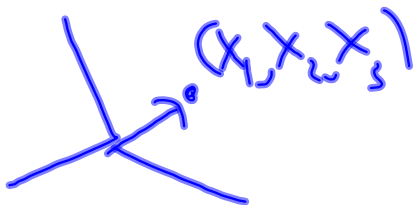
Ex. $z = x + iy$ a complex number



In fact: $|x+iy| = \sqrt{x^2+y^2}$

More generally: In Euclidean n -space,

$$|(x_1, x_2, \dots, x_n)| = (x_1^2 + x_2^2 + \dots + x_n^2)^{\frac{1}{2}}$$



Problem: A substance is 99% water.

Some water evaporates, leaving a substance that is 98% water.

How much water evaporated?

→
What percentage of original water evaporates?