407, 8/22

15 to 20

* Attendance
* Physical accessibility announcement
* Go over syllabus (10 mins) → Importance of: reading, individual HW
* Pass out group HW teams.
  ↳ group introductions, sit together...

Mathematics

HS model: Concepts, ideas, definitions → Problem → Solution

reality: Problem → ideas, definitions, concepts → solutions (often, may have different sols.)

* Good mathematics requires good definitions.
  → Teaches must be sensitive to alternatives!

Ex: For a real number x, how would you define $|x|$? (absolute value of x)?

[$\mathbb{R}$ = set of real #'s = ←——0——→]

Possible defs: 1) $|x| = \begin{cases} x & \text{if } x>0 \\ -x & \text{if } x<0 \\ 0 & \text{if } x=0 \end{cases}$

2) $|x|$ = dist from 0 on #-line,

3) $|x| = \sqrt{x^2}$,

Fact: These defs are logically equivalent.

However: Some better than others in applications.

Ex: Key props of $|\cdot|$

a) $|x| \geq 0$ and $|x| = 0$ iff $x = 0$ (positive definiteness)

b) $|xy| = |x||y|$ (multiplicativity)

c) $|x+y| \leq |x| + |y|$ (subadditivity)

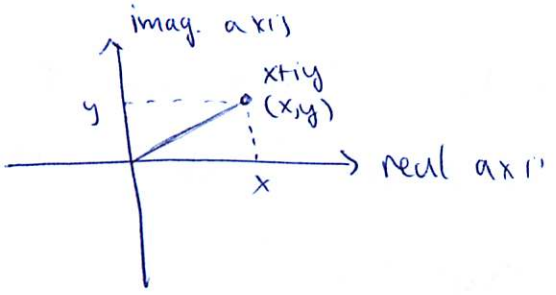
d) $|x| \leq y$ iff $-y \leq x \leq y$ (for $y > 0$)

e) $|-x| = |x|$

Property	Def making prop evident
a	2
b	3
c	2
d	2
e	2

* The most important thing about good defns:
Ability to generalize.

For a complex number, $z = x+iy$,



What is $|z|$?

Possible defs: $|z| = \sqrt{z^2}$ (not so good...)

Likewise, def 1) is bad because "positive" and "negative" don't generally apply.

Instead: $|z|$ = distance from z to origin

$= \sqrt{x^2+y^2}$

More generally, in Euclidean n-dimensional space,

$$|(x_1, \dots, x_n)| := \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

= distance from $(x_1, \dots, x_n)$ to 0.

Q: How about

$$|(x_1, \dots, x_n)|_p := \left(|x_1|^p + |x_2|^p + \dots + |x_n|^p \right)^{1/p}$$

$\rightsquigarrow$ called "L^p norm"

In real life: Driving in Manhattan, shortest distance is L^1 norm.

Problem Analysis:

* A substance is 99% water. Some water evaporates, leaving a substance that is 98% water. How much water has evaporated?

Sol: Since there are 100 units of mixture to start.

100 total { 99 units water, 1 unit "stuff" } —water evaporates→ { x units water, 1 unit stuff }

$$0.98 = \frac{x}{1+x} \quad \text{so} \quad 0.98 + 0.98x = x$$

$$\Rightarrow \quad 0.98 = 0.02x$$

$$\Rightarrow \quad x = 49$$

So 50 units of H_2O evaporated, or $\frac{50}{99} = 0.505\ldots$% of the mix!

→ SURPRISING!! Want to better understand.

Numerical sol: percent "stff" goes from 1% to 2%, i.e., dubles ⟹ total of water plus stuff is cut in half (as amt stff unchanged)

so 50 units total, hence 49 units of H_2O.