Q: What is an equation?

		VOTES	
(1)	x^2+4x-3	0	(no = sign)
(2)	$3x-2=7$	13	
(3)	$7=5+2$	13	
(4)	$1=13$	8	(no: not true)
(5)	$2x+1=x+(x+1)$	13	
(6)	$1+x=13+x$	8	

Can think of eqn. as a statement of equality, always part of a longer sentence.

$\longrightarrow$ Can be T or F

Ex: There exists $x \in \mathbb{R}$ such that

$$3x-2=7$$

- The number $x=2$ is not a sol'n of $3x-2=7$
- As a statement about real numbers, $1=13$ is false.

Three fundamental features of $=$

(1) $a=a$ (reflexive)

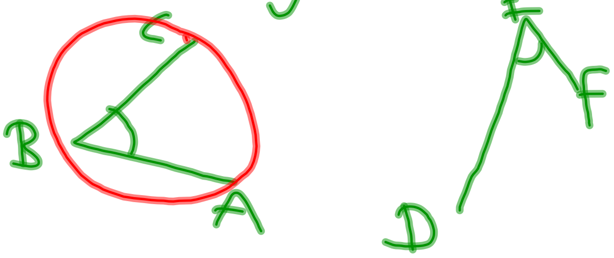
(2) $a=b$ then $b=a$ (symmetric)

(3) If $a=b$ and $b=c$, then $a=c$ (transitive)

Def: Any relation that satisfies 1–3 is called an equivalence relation

Ex: $\Longleftrightarrow$ (iff) is an equivalence relation on T/F statements.

- Geometry: Congruent angles



$\angle ABC \cong \angle DEF$ means $m\angle ABC = m\angle DEF$ as actual #'s

- **Fractions:** $\frac{4}{2} = \frac{2}{1}$ (It's better to say "$\frac{4}{2}$ is equivalent to $\frac{2}{1}$")

Isomorphism

$\underbrace{\text{Iso}}_{\text{same}}\underbrace{\text{morphism}}_{\text{structure}}$

Ex: Recall from last time

$$(\mathbb{R},+) \longleftrightarrow (\mathbb{R}_{>0}, \times)$$

$$p+q \longleftrightarrow ab$$

$$0 \longleftrightarrow 1$$

More precisely, for any real # $e>1$

1-1 correspondence.

$$(\mathbb{R},+) \longleftrightarrow (\mathbb{R}_{>0}, \times)$$

$$p \longmapsto e^p$$

$$\log_e(a) \longleftarrow\!\!| \; a$$

$$p+q \longmapsto e^{p+q} = e^p \times e^q$$

$$= \log_e(ab) \longleftarrow\!\!| \; ab$$

$$\log_e(a) + \log_e(b)$$

Def: Two mathematical structures are isomorphic if there is a 1-1 correspondence b/w them s.t. operations in one structure give answer that corresponds to the operation in other structure.

Ex: $S = \{(a,b) \; \vdots \; a, b \in \mathbb{Z}, \; b \neq 0\}$

such that

and we consider (a,b) and (c,d) to be <u>equivalent</u> if $ad = bc$

<u>Define</u> $(a,b) \otimes (c,d) := (ac, bd)$

LHS is <u>defined</u> to be RHS

$(a,b) \oplus (c,d) := (ad+bc, bd)$

<u>Ex</u> $(4,-1) \oplus (3,2) = (5,-2)$

$(3,1) \otimes (1,3) = (3,3) = (1,1)$

<u>Claim</u>:

$S, \oplus, \otimes$ is isomorphic to $\mathbb{Q}, +, \times$ via $(a,b) \mapsto \frac{a}{b}$