8/29 → Annoncement about group re-organization PROBS: p140 #5

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* HS students often think of algebra as little more than techniques. However, there are important underlying <u>ideas</u> in alg, and understanding these will make you better at teaching and doing algebra

<u>Q</u>: What is an equation?

(1) $x^2 + 4x - 3$

(2) $3x - 2 = 7$

(3) $7 = 5+2$

(4) $1 = 13$

(5) $2x + 1 = x + (x+1)$

(6) $1 + x = 13 + x$

* Ask for vote, ~~then formal defn~~
Test consistency of ~~defn~~ opinions against ans.

Can think about an eqn as a phrase belonging in a bigger sentence: Can be T/F and part of many diff. sentences!

e.g. • $\exists\ x \in \mathbb{R}$ such that $3x - 2 = 7$

• The number $x = 2$ is not a solution to $3x - 2 = 7$

• As a statement about real numbers, $1 = 13$ is false

• $\forall\ x,\quad 2x + 1 = x + (x+1)$.

* Have groups discuss def of eqn, and write on board.

We'll settle on: Def: An **equation** is a sentence / statement of equality between two expressions.

Example: (next time?)

Solving eqns: Def: The **solutions** of an equation are all values of the variables which make the eqn. true.

Ex: The solutions of $3x - 2 = 7$ are $x = 3$.

$\Updownarrow$

3 is a solution to $3x - 2 = 7$ because $3(3) - 2 = 7$

3 is the only sol. to $3x - 2 = 7$ because:

if $3x-2=7$,

then $(3x-2)+2 = 7+2$ (adding the same number to = expressions preserves =)

then $3x = 9$ (assoc, add inv, add iden)

then $\frac{3x}{3} = \frac{9}{3}$ (dividing = expressions by 3 yields = results)

then iff $x = 3$.

so $x=3$ is the <u>only</u> solution.

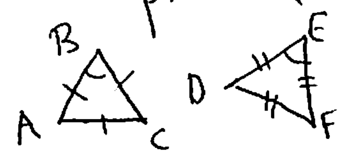
<u>Key props of =</u>

- For any x, $x=x$ holds (reflexive)
- If $x=y$ then $y=x$ (symmetric)
- If $x=y$ and $y=z$, then $x=z$ (transitive)

Def: Any relation which is reflexive, symmetric, & transitive is called an <u>equivalence relation</u>

Ex. 1) iff ($\Leftrightarrow$) is an equivalence rel'n because if p, q, r are statements then:
- $p \Leftrightarrow p$
- If $p \Leftrightarrow q$ then $q \Leftrightarrow p$
- If $p \Leftrightarrow q$ and $q \Leftrightarrow r$, then $p \Leftrightarrow r$.

2) Congruence of angles: Two angles $\angle ABC$, $\angle DEF$ are congruent provided
$m\angle ABC = m\angle DEF$
their measures are =.



3) Fractions: Consider the set $S = \{(a,b) \mid a, b \in \mathbb{Z}, b \neq 0\}$ of ordered pairs of integers with $b \neq 0$.

We define an equivalence relation on S as follows:

$$(a,b) \sim (c,d) \quad \text{iff} \quad a \cdot d = b \cdot c$$

($\sim$ ↑ is equivalent to)

Ex. $(3,1) \sim (6,2)$
$(4,5) \sim (28,35)$

- Symmetric: $(a,b) \sim (a,b)$ because $ab = ab$
- reflexive: If $(a,b) \sim (c,d)$ then $ad = bc \Leftrightarrow cb = da$ $\Rightarrow (c,d) \sim (a,b)$
- trans.....

Does this equivalence relation look familiar?

So ~~equality~~ equivalence is a <u>weaker</u> notion of equality.

<u>Isomorphism</u>

Recall from last time: correspondence

$$(\mathbb{R}, +) \longleftrightarrow (\mathbb{R}_{>0}, \times)$$

$$p+q \longleftrightarrow ab$$

$$0 \longleftrightarrow 1$$

$$\vdots$$

More precisely, consider the correspondence, for any $e>1$

$$(\mathbb{R}, +) \longleftrightarrow (\mathbb{R}_{>0}, \times)$$

$$p \longmapsto e^p$$

$$\log_e(a) \longleftarrow a$$

<u>Observe</u>: $p+q \longmapsto e^{p+q} = e^p \cdot e^q$

$$\log_e(a) + \log_e(b) = \log_e(ab) \longleftarrow ab$$

<u>Def</u>: ~~An isomorphism is~~ Two mathematical structures are <u>isomorphic</u> if there is a 1-1

correspondence between them such that operations in one structure give answers corresponding to operations in the other structure.

<u>Ex</u>: $S = \{(a,b) \mid a,b \in \mathbb{Z},\ b \neq 0\}$, and we consider $(a,b) \sim (c,d)$ iff $ad = bc$

We define
$$(a,b) \oplus (c,d) = (ad+bc,\ bd)$$
$$(a,b) \otimes (c,d) = (ac,\ bd)$$

~~Then $(a,b) \mapsto \frac{a}{b}$ defines an isomorphism~~

Then S is isomorphic to $\mathbb{Q}$ = rationals

via
$$(a,b) \longmapsto \frac{a}{b}$$
$$\otimes \longmapsto \times$$
$$\oplus \longmapsto +$$