

Def.: The sols of an equation are all values of the variables making the eqn. true.

Ex.: Let's solve $3x-2=5$

$$\begin{aligned}3x-2 &= 5 \\3x &= 7 \\x &= \frac{7}{3}\end{aligned}$$

1) $x = \frac{7}{3}$ is a sol.

to $3x-2=5$

2) $x = \frac{7}{3}$ is the only sol.

1) $x = \frac{7}{3}$ is a solution because
 $3\left(\frac{7}{3}\right) - 2 = 7 - 2 = 5$

2) If x is a solution to $3x - 2 = 5$
then $3x = 7$ because $\dots\dots$
then $x = \frac{7}{3}$ because $\dots\dots$
Hence $x = \frac{7}{3}$ is the only sol. to $3x - 2 = 5$.

* Equations are part of a larger sentence.

Ex: \exists a real number x such that
 $3x-2=5$

there
exists

existential statement

TRUE: we proved $x = \frac{7}{3}$ works.

Ex: \exists an integer x such that
 $3x-2=5$

FALSE: we proved $x = \frac{7}{3}$ is
the only sol to $3x-2=5$ and $\frac{7}{3} \notin \mathbb{Z}$.

Ex: \forall real numbers x , $3x-2=5$

for all
for every

FALSE: b/c \exists a
real number x such that
 $3x-2 \neq 5$.

Quantifiers: \exists , \forall

Def: An identity is an eqn.
which is true for all (\forall) of
the variables.

Ex: $x^2-9=(x-3)(x+3)$
 $\forall x \rightarrow \sin^2(x) + \cos^2(x) = 1$

* Solving equations = process of mathematical reasoning.

Q. How do you respond to:

Solve $x^2 - 3x - 4 = 0$

Why move the 4 to other side?

$$x^2 - 3x = 4$$
$$x(x-3) = 4$$
$$x=2 \text{ or } x-3=2$$
$$x=2 \text{ or } x=5$$

look at reasoning!

Mis-understanding of "zero principle"
If $AB=0$ then $A=0$ or $B=0$

Does NOT hold!

Observe: $x=2, x=5$ do not satisfy orig. eqn.

Zero Principle: If A and B are real numbers satisfying $AB=0$, then $A=0$ or $B=0$.

Pf. If $AB=0$ and $A \neq 0$ then \exists a real number A^{-1} s.t. $A^{-1} \cdot A = 1$. Hence,

$$(A^{-1}A)B = A^{-1}(AB) = A^{-1} \cdot 0 = 0$$

\parallel Thus, $A \neq 0 \Rightarrow B=0$

\parallel Here, since either

\parallel $A=0$ or $A \neq 0$

Conclude $A=0$ or $B=0$

crucial use of 0
 $x \cdot (0+0) = x \cdot 0 + x \cdot 0$
 $\parallel \leftarrow 0 = + \text{ idem}$
 $x \cdot 0$
 $\parallel \Rightarrow x \cdot 0 = 0$

Correct sol'n: $x^2 - 3x - 4 = 0$

$(x+1)(x-4) = 0$

$x+1=0$ OR $x-4=0$

$x=-1$ OR $x=4$.

1) Mathematically justify

? 2) Motivate this reasoning?

$\forall x$
 $(x+1)(x-4) = x^2 - 3x - 4$

"FOIL": Dist. prop (2 times)