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Def: The <u>solutions</u> of an equation are all values of the variables making the equation true.

Ex: Let's solve $3x-2=5$.

$$3x-2=5$$
$$3x=7$$
$$x=\frac{7}{3}$$

* $x=\frac{7}{3}$ is a solution to $3x-2=5$, and this is the <u>only</u> solution.

<u>Two</u> things to <u>prove</u>:

1) $x=\frac{7}{3}$ is a solution to $3x-2=5$ because

$3\left(\frac{7}{3}\right)-2 = 7-2 = 5$

2) If x_0 is a solution to $3x-2=5$,

then $3x=7$ because ....

then $x=\frac{7}{3}$ because ...

So x a solution to $3x-2=5$

$\Downarrow$

$x=\frac{7}{3}$. Hence, this is the only sol.

* Equations are a phrase in a larger sentence, and this sentence is important!

Ex $\exists$ a real number x such that $3x-2=5$

there exists

Example of an "existential statement"

We just showed the above statement is TRUE, because $x=\frac{7}{3}$ satisfies $3x-2=5$

Ex: $\exists$ an integer x such that $3x-2=5$

This existential statement is FALSE, because we showed the ONLY solution to $3x-2=5$ is $x=\frac{7}{3}$, which is not an integer.

⤳ Domain ($:=$ set of possible values for variable) is important!

Ex: $\forall$ numbers x, $3x-2=5$

for all

This is FALSE if the domain is $\mathbb{R}$ or $\mathbb{Z}$ or... because there are values of x not satisfying the equ_

Def: An *identity* is an equation which is true for all values of the variables.

Ex: $\underbrace{\forall \text{ real numbers } x, \quad x^2-3x+2 = (x-2)(x-1)}_{\text{example of universal statement}}$

* $\exists, \forall$ are called *quantifiers*. It is very important to keep track of them!

** Solving equations = process of mathematical reasoning

$\to$ So *must* know precisely what statements are being made..

Q: How would you respond to the following:

Solve $x^2-3x-4=0$.

"Ans":

$$x^2-3x=4$$
$$x(x-3)=4$$
$$x=2 \quad \text{or} \quad x-3=2$$
$$x=2 \quad \text{or} \quad x=5$$

look at reasoning in each step!

[5 mins of thought]

Correct Solution

$$x^2 - 3x - 4 = 0$$

$$(x+1)(x-4) = 0$$

$$x+1=0 \quad \text{OR} \quad x-4=0$$

$$x=-1 \quad \text{OR} \quad x=4$$

Q. How to justify steps?

Q: How to help students see difference between correct / incorrect steps?

First: What are we trying to do?

a) Trying to prove that $x^2-3x-4=0$

b) Trying to show that if $x^2-3x-4=0$, then $x=-1$ or $x=4$

c) Trying to show that if $x=-1$ or $x=4$, then $x^2-3x-4=0$

d) Both b) and c).

* In groups prove that the solutions to $x^2-3x-4=0$ are exactly $x=-1$, $x=4$. } with all reasoning, if's, and's etc!

Crucial tool:

Zero factor principle

Suppose that $A \cdot B = 0$. Then either $A = 0$ or $B = 0$ (or both!)

proof. Suppose that $A \cdot B = 0$.

If $A \neq 0$, then $\exists\, A^{-1}$ such that $A^{-1} \cdot A = 1$

So multiplying both sides of $AB = 0$ by A^{-1} gives

$$A^{-1}(AB) = A^{-1} \cdot 0$$

$$(A^{-1}A)B = 0 \qquad [\text{assoc}, \ x \cdot 0 = 0]$$

$$1 \cdot B = 0 \qquad [\text{mult inv}]$$

$$B = 0 \qquad [\text{mult iden}]$$

So $A \neq 0 \implies B = 0$.

Since either $A = 0$ or $A \neq 0$, we conclude either $A = 0$ or $B = 0$. ∎