Problems: p. 159 #4, 7

Last time: Solving linear eqns

$\longrightarrow$ Graphs

Today: Solving quadratic eqns

$\longrightarrow$ Complex nos.

* Find two numbers m, n whose sum is 10 and whose product is 12.

$$\begin{cases} m+n=10 \\ mn=12 \end{cases} \longrightarrow m=\frac{12}{n} \text{ so } \frac{12}{n}+n=10$$

$$\frac{12}{n} + n = 10$$

Since $n \neq 0$, this is equivalent to

$$12 + n^2 = 10n \quad \text{or}$$

$$n^2 - \underset{\substack{\uparrow \\ -\text{sum}}}{10}n + \underset{\substack{\uparrow \\ \text{prod}}}{12} = 0$$

Observe

Also, as $\begin{cases} m+n=10 \\ mn=12 \end{cases}$ is symmetric in m, n.

$$m^2 - 10m + 12 = 0$$

Q: A) How do you solve this eqn?
B) Why does your method work?

A) · Quadratic Formula
- · (Graphing)
- – Completing the $\square$
- – Factoring

B) · Prove quadratic formula
- – that
- – why

Possible Solution:

Since $m+n=10$, the average of m,n is 5. So, we can write

$$m = 5 + x$$
$$n = 5 - x$$

Then $mn = (5+x)(5-x) = 12$

$$= 25 - x^2 = 12$$

So $x^2 = 13$ and $x = \pm\sqrt{13}$

$$\Longrightarrow \quad \begin{aligned} m &= 5+\sqrt{13} \\ n &= 5-\sqrt{13} \end{aligned} \quad \text{or vice versa}$$

Thm: If m,n are solutions to $x^2+bx+c=0$ then $m+n=-b$ and $mn=c$

and conversely

Pf: ($\Longrightarrow$) Since m, n are solutions to $x^2+bx+c=0$, we have

as quadratic polys.

$$x^2+bx+c = (x-m)(x-n)$$

$$= x^2 - mx - nx + mn$$

$$= x^2 - (m+n)x + mn$$

$$\Longrightarrow \quad b = -(m+n)$$
$$c = mn$$

Let's use this idea to solve

$$x^2+bx+c=0$$

Let m,n be the solutions.

So $m+n=-b$

$mn=c$

Average of m,n is $\frac{-b}{2}$

So can write

$$m=\frac{-b}{2}+v \leftarrow \text{deviation from average}$$

$$n=\frac{-b}{2}-v$$

Then $c=mn=\left(-\frac{b}{2}+v\right)\left(-\frac{b}{2}-v\right)$

$$=\frac{b^2}{4}-v^2$$

Hence $v=\pm\sqrt{\frac{b^2}{4}-c}$

So $\underbrace{m=-\frac{b}{2}+\sqrt{\frac{b^2}{4}-c}}_{=\frac{-b+\sqrt{b^2-4c}}{2}}$, $n=-\frac{b}{2}-\sqrt{\frac{b^2}{4}-c}$

or vice versa

To solve $ax^2+bx+c=0$,

if $a\neq 0$, just solve

$$x^2+\underbrace{\frac{b}{a}}_{B}x+\underbrace{\frac{c}{a}}_{C}=0 \Bigg\} \frac{-B\pm\sqrt{B^2-4C}}{2}$$

$$=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

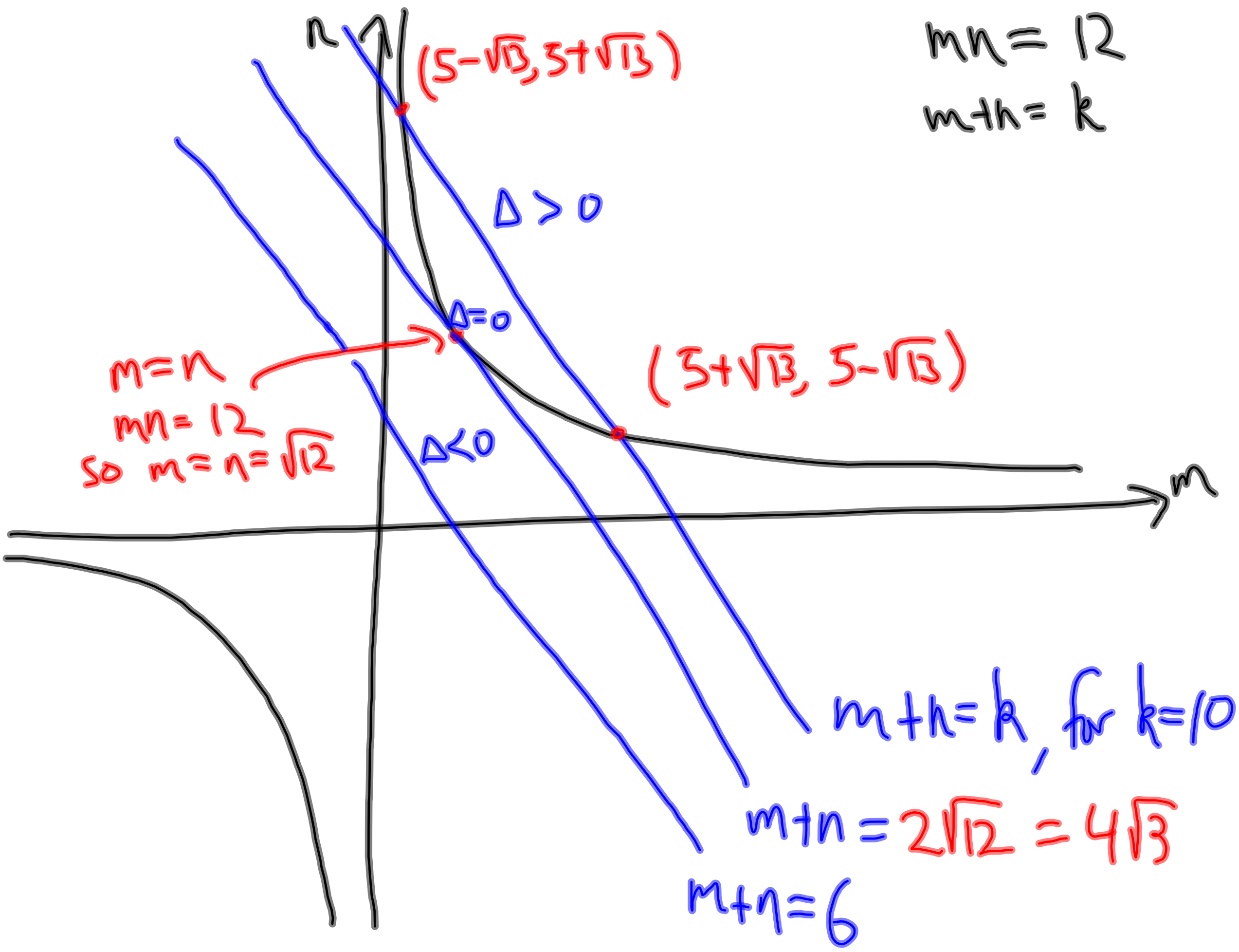
Def: b^2-4ac is called the discriminant.

Cor: For a,b,c real numbers, $a\neq 0$,

ax^2+bx+c has $\begin{cases} 2 \\ 1 \\ 0 \end{cases}$ distinct real roots

according to as $\begin{cases} \Delta > 0 \\ \Delta = 0 \\ \Delta < 0 \end{cases}$, $\Delta = b^2-4ac$.

$\left(\dfrac{-b\pm\sqrt{\Delta}}{2a}\right)$



Q: $\mathbb{C} = \{a+bi \mid a, b \in \mathbb{R}\}$

Add, multiply as usual, using $i^2 = -1$

Thm: Any poly w/ $\mathbb{C}$-coeffs has at least one complex root.

Can you find a solution to $x^4 = -1$?