Problems: p 163, #3

Q: What are some of the "standard" rules for solving equations? For each rule, give an ex. of it in action.

* Isolate the variable. $x+2=2x-3 \rightsquigarrow x=5$
* Combine like terms: $2x+3x=5x$
* Don't divide by 0.
* PEMDAS : $3(2x+4\div 7-6x)+34\times 6$
* Show all your work
* Add/subtract same expression from both sides.

$$x+1=2$$
$$x+1-1=2-1$$
$$x=1$$

* multiply/divide ________________
* Apply same function to both sides. $3=\log_2(x)$

  Apply 2^{*} to both sides: $2^3=2^{\log_2(x)}$

  $=x$

* Check your work.

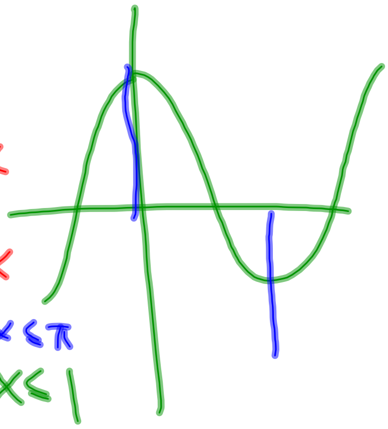
We forgot: approximate methods

- graph
- tables

Two key points: i) Vast majority of equations "in life" can't be solved exactly! State this, then say we focus on ones we can solve.

ii) The above rules must be applied with care!

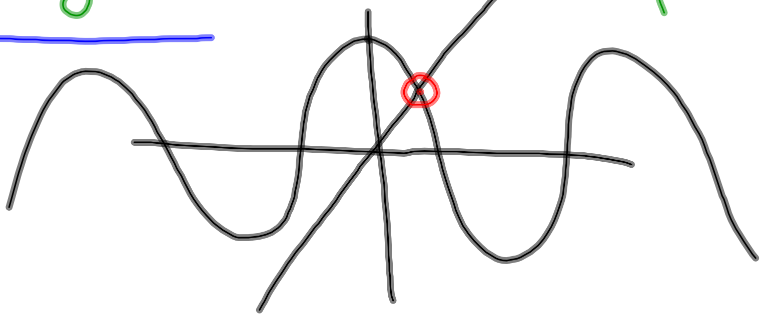
Ex: Solve $x = \cos(x)$



- Isolate var: $0 = \cos(x) - x$

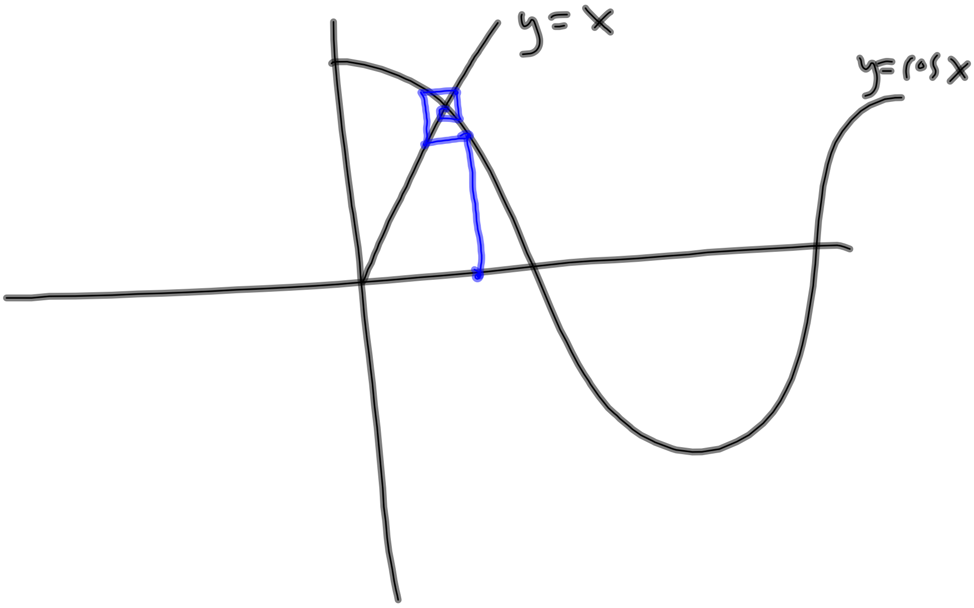
$$0 \le x \le 1 \left\{ \cos^{-1}(x) \overset{\text{true only for } -1 \le x \le 1}{=} \cos^{-1}(\cos(x)) \underset{\text{for } 0 \le x \le \pi}{=} x \right.$$

$\cos^{-1}(x)$ defined for $-1 \le x \le 1$, taking values $0 \le \cos^{-1}(x) \le \pi$



$$\begin{aligned} x &= \cos(x) \\ &= \cos(\cos(x)) \\ &= \cos(\cos(\cos(x))) \end{aligned}$$

Take random number x, $0 \le x \le \pi/2$ and compute $\cos(\cos(\cos(\cdots \cos(x))))$



Ex: Find all sols to

$$x + \log(3-x) = 4 + \log(3-x)$$
$$\underline{-\log(3-x) \qquad -\log(3-x)}$$
$$x = 4$$

Problem! $x=4$ is not a sol of the original eqn. b/c. $\log(-1)$ is not defined.

Thm: Let f, g, h be functions.

For all x in the intersection

* of the domains of f, g, & h,

$$f(x) = g(x) \iff f(x) + h(x) = g(x) + h(x)$$

Ex: Solve

$$\frac{6t-2}{t^2-1} + \frac{t-3}{t-1} = 0$$

$$\iff \frac{6t-2}{(t-1)(t+1)} + \frac{t-3}{t-1} = 0$$

$$\iff \frac{6t-2+(t+1)(t-3)}{(t-1)(t+1)} = 0$$

$$\implies 6t-2+(t+1)(t-3) = 0$$

for $t=1$ is div by 0

$$\iff 6t-2+t^2-2t-3 = 0$$

$$\iff t^2+4t-5 = 0$$

$$\iff (t+5)(t-1) = 0$$

$$\iff \text{So } t=-5 \text{ or } t=1$$

Thm: If f, g, h are functions, then for all x in the intersection of the domains of f, g, h, with $h(x) \neq 0$.

$$f(x) = g(x) \iff f(x) \cdot h(x) = g(x) \cdot h(x)$$

Ex: Thm: If $1 = 2$ then $x = y$ for real numbers x, y.

pf $1 = 2 \Rightarrow 1 - 1 = 2 - 1$

$0 = 1$

$\Rightarrow x \cdot 0 = x \cdot 1$

$0 = x, \ \forall x$

Thus, $\forall x, y$, $x = 0$ and $y = 0$

so $x = 0 = y$.

Q: Solution to $x^4 = -1$?

$x = \sqrt[4]{-1}$

$i = \sqrt{-1}$

$x = \pm\sqrt{i}$

$x^2 = i$

$a, b \in \mathbb{R}$ $\quad (a+bi)^2 = i$

$\qquad (a^2-b^2) + 2abi = i = 0 + 1\cdot i$

so $a = \pm b$
and $2ab = 1$
so $2a^2 = \pm 1$
$a^2 = \pm \frac{1}{2}$
$a = \pm\sqrt{\pm \frac{1}{2}}$

Since $a, b \in \mathbb{R}$, $\quad a = \pm\sqrt{\frac{1}{2}}$
$\qquad b = \pm\sqrt{\frac{1}{2}}$

i^i

$x = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$

Satisfies $x^4 = -1$

(So does $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i$, $-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i$, $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$