Problems: p 166, #2

Often, you'll recognize an eqn. has the form

$$h(f(x)) = h(g(x))$$

for f, g, h functions.

Temptation: Apply h^{-1} to both sides

$$f(x) = g(x)$$

We have 2B careful.

Ex: (1) $\sqrt{3x-2} = -\sqrt{4x-3}$

Naive: $\Downarrow$ $\left(\sqrt{3x-2}\right)^2 = \left(-\sqrt{4x-3}\right)^2$

$$3x-2 = 4x-3$$

$$\Leftrightarrow \quad 1 = x$$

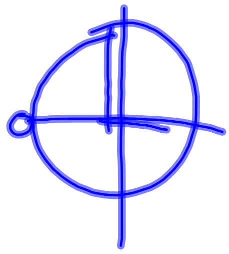
But $x=1$ is not a sol, as

$$\sqrt{1} \neq -\sqrt{1}$$

$$1 \neq -1$$

(2) $\cos^{-1}(x) = \cos^{-1}(x-2) + \pi$

Apply cos to both sides



$$\cos(\cos^{-1}(x)) = \cos(\cos^{-1}(x-2)+\pi)$$

$$\underset{\text{addition formula}}{=} \cos(\cos^{-1}(x-2))\cdot\cos(\pi) - \sin(\cos^{-1}(x-2))\cdot\overset{0}{\overset{\shortparallel}{\sin(\pi)}}$$

$$x = (x-2)\cdot(-1)$$

So $x = 2-x$, i.e. $x=1$

But: $x=1$ is NOT a sol to orig eqn:

$$\cos^{-1}(1) \overset{?}{=} \cos^{-1}(1-2)+\pi \overset{?}{=} \underbrace{\cos^{-1}(-1)+\pi}_{\shortparallel}$$

$$\cos^{-1}(1) = 0 \neq \pi+\pi = 2\pi$$

(3) Sometimes, things are ok:

$$\sqrt[3]{3x-2} = -\sqrt[3]{4x-3}$$

cube both sides

$$3x-2 = -(4x-3)$$
$$7x = 5$$
$$x = 5/7$$

Indeed:

$$\sqrt[3]{3\left(\frac{5}{7}\right)-2} \overset{?}{=} -\sqrt[3]{4\left(\frac{5}{7}\right)-3}$$

$$\sqrt[3]{\frac{15-14}{7}} \overset{?}{=} -\sqrt[3]{\frac{20-21}{7}}$$

i.e. $\sqrt[3]{1/7} \overset{?}{=} -\sqrt[3]{-1/7}$

YES!!

Q What is Going on?

* How are the graphs of $y=x^2$ and $y=x^3$ <u>fundamentally</u> different?

<u>Recall</u>: A function $h(x)$ is called "1-1" if

one to one

def of fn function

$$h(x)=h(y) \Longleftrightarrow x=y$$

$\xRightarrow{\text{1-1}}$

<u>Graphically</u>

"h can't cross any horiz line more than once."

Thm: Let $h(x)$ be a 1-1 function. For all x in the domains of f and g for which $f(x), g(x)$ are in the domain of h,

$$h(f(x)) = h(g(x))$$
$$\Updownarrow$$
$$f(x) = g(x)$$

proof: Since h is a function,

$$f(x) = g(x) \Longrightarrow h(f(x)) = h(g(x))$$

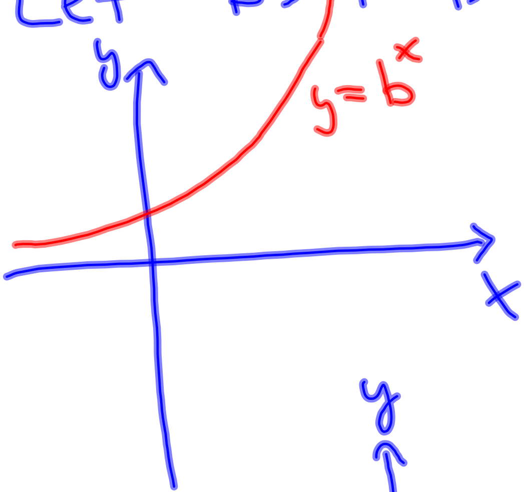
Conversely, if $h(f(x)) = h(g(x))$ then, since h is 1-1,

$$f(x) = g(x).$$

Application: Two "classic" 1-1 functions:

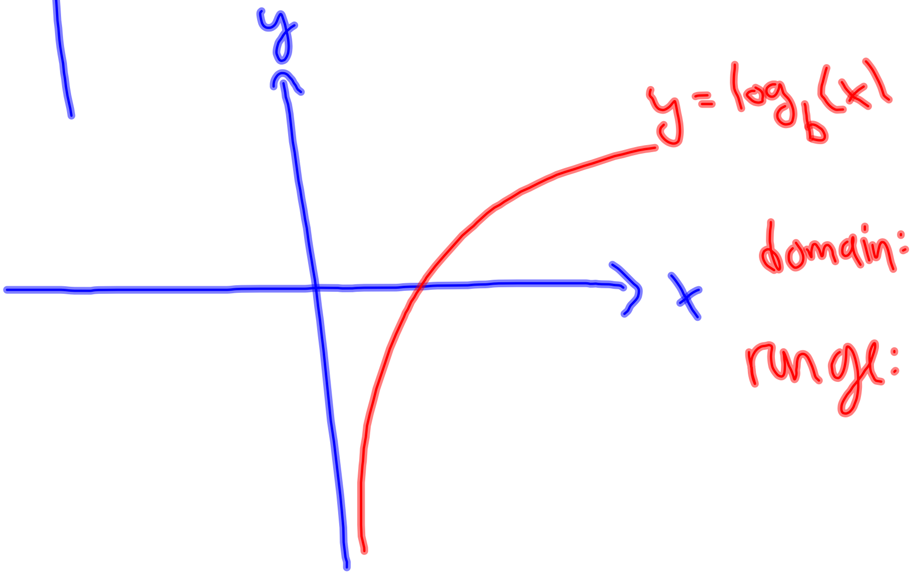
exponentials, logarithms.

Let $b > 1$ be a real number.



domain: $(-\infty, \infty)$

range: $(0, \infty)$



domain: $(0, \infty)$

range: $(-\infty, \infty)$

<u>Moreover</u>: $\log_b(b^x) = x$ (for $x \in (-\infty,\infty)$)

$b^{\log_b(x)} = x$ (for $x \in (0,\infty)$)

Ex: Solve

$$2\log_2(x+3) = \log_2(10x+14)$$

$$\log_2((x+3)^2) = \log_2(10x+14)$$

So

$$2^{\log_2((x+3)^2)} = 2^{\log_2(10x+14)}$$

using $y = \log_2(x)$ is 1-1.

$$(x+3)^2 = 10x+14$$

$$x^2+6x+9 = 10x+14$$

$$x^2-4x-5=0$$

$$(x-5)(x+1)=0$$

So $x=5$ or $x=-1$.

#5, p166

$L :=$ linear functions on $\mathbb{R}$

$= \left\{ f(x) = ax+b \;\middle|\; \begin{array}{l} a,b \in \mathbb{R} \\ a \neq 0 \end{array} \right\}$

This set has a rule of composition: composition of functions

$(f \circ g)(x) := f(g(x))$

- L is closed under $\circ$

$(ax+b)\circ(cx+d) = a(cx+d)+b$

$= (ac)x + (ad+b)$

- $\circ$ is assoc.
- $id(x) = x$ is the identity function
- If $f(x) = ax+b$, then

$f^{-1}(x) = \frac{1}{a}x - \frac{b}{a}$ because

$f(f^{-1}(x)) = a\left(\frac{1}{a}x - \frac{b}{a}\right) + b = x = id(x)$

$f^{-1}(f(x)) = \frac{1}{a}(ax+b) - \frac{b}{a} = x = id(x)$

So L is a group!

Ex: Solve $5x+14 = 4$

$f(x) = 4$

$\overset{\shortparallel}{5x+14}$

$f^{-1}(x) = \frac{1}{5}x - \frac{14}{5}$

Apply f^{-1} to both sides

$x = f^{-1}(f(x)) = f^{-1}(4) = \frac{4}{5} - \frac{14}{5} = \frac{-10}{5}$

$= -2.$