Problems: p176-7, #4, 11.

Prob. A: On a round trip, you average 30mph on way out 60mph on way back.

? What is your ave speed for whole trip?

Prob B): On a trip, you ave 30mph for a certain time, then I take over & average 60mph for same amount of time.

What is our ave. speed for the total time?

(A) 30mph → 60mph ←

$$30\left(\frac{2}{3}\right) + 60\left(\frac{1}{3}\right)$$

$$= 40 \text{ mph}$$

B) 30mph 60mph

$$\frac{60+30}{2} = \frac{90}{2} = 45 \text{ mph}$$

A) d = total dist.

$$\text{ave speed} = \frac{\text{ttl. dist}}{\text{ttl. time}}$$

$$30 = \frac{d}{\text{time @ 30 mph}} \qquad \text{time @ } 30 = \frac{d}{30}$$

$$60 = \frac{d}{\text{time @ 60mph}} \qquad \text{time @ } 60 = \frac{d}{60}$$

$$\left.\begin{array}{l} \text{Total time} = \frac{d}{30} + \frac{d}{60} \\ \text{Total dist} = 2d \end{array}\right\} \text{ave speed} = \frac{2d}{\frac{d}{30} + \frac{d}{60}}$$

$$= \frac{2}{\frac{1}{30} + \frac{1}{60}} = \frac{2 \cdot 60}{2 + 1} = \frac{120}{3} = 40$$

$$= \frac{2 \cdot 30 + 60}{3}$$

B) ave speed $= \dfrac{\text{ttl. dist}}{\text{ttl. time}}$

Suppose that we each drive for t-hours.

dist. traveled @ 30mph = 30t

——— @ 60mph = 60t

ttl dist = 30t + 60t

ttl time = 2t

ave speed $= \dfrac{30t + 60t}{2t}$

$= \dfrac{30 + 60}{2} = 45$

* dist = speed · time

time $= \dfrac{\text{dist}}{\text{speed}}$

A) Same problem, <u>but</u> speeds v, w

B) ——————————— v, w

A $\rightarrow$ time @ v mph $= \frac{d}{v}$, time @ w mph $= \frac{d}{w}$ $\Big\}$ ttl time $= \frac{d}{v} + \frac{d}{w}$, ttl dist $= 2d$

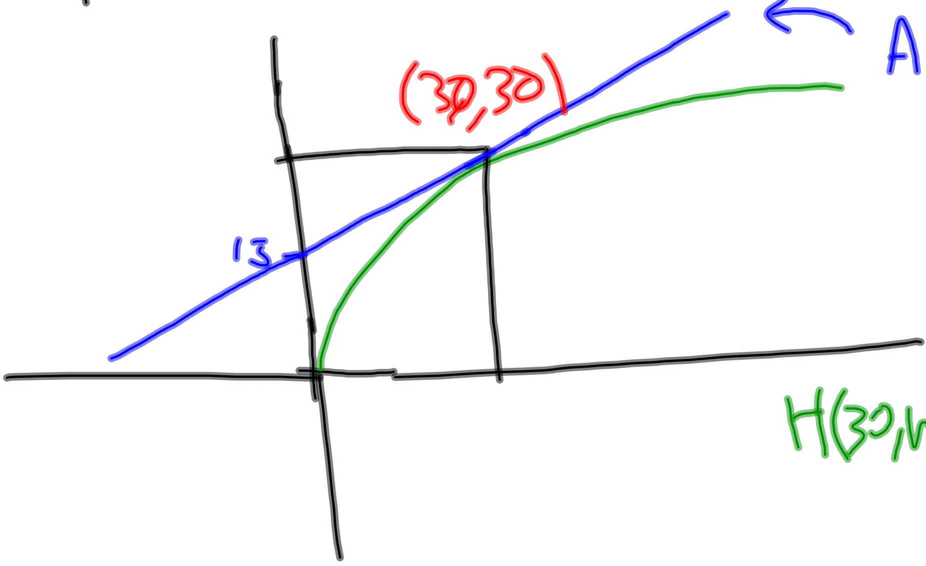
So ave speed $= \dfrac{2d}{\frac{d}{v} + \frac{d}{w}} = \dfrac{2}{\frac{1}{v} + \frac{1}{w}} =: H(v,w)$

B $\rightarrow$ ave speed $= \dfrac{v+w}{2} =: A(v,w)$

$$\left[* \quad \frac{1}{H(v,w)} = \frac{\frac{1}{v} + \frac{1}{w}}{2} = A\left(\tfrac{1}{v}, \tfrac{1}{w}\right) \right]$$

Comparison of $H(v,w)$, $A(v,w)$:

Fix $v=30$, graph $H(30,w)$ as $A(30,w)$ functions of w.



$$A = \frac{30+w}{2} = 15 + \frac{w}{2}$$

$$H(30,w) = \frac{2}{\frac{1}{30}+\frac{1}{w}} = \frac{60w}{30+w}$$

Thm: $H(v,w) \leq A(v,w)$, w/ = iff $v=w$
(for $v,w>0$)

Def: The harmonic series:

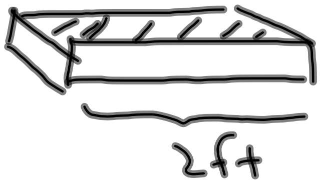
$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots, \frac{1}{n},$$

$$H\left(1, \tfrac{1}{3}\right) = \frac{2}{\frac{1}{1} + \frac{1}{\frac{1}{3}}} = \frac{2}{4} = \frac{1}{2}$$

$$H\left(\tfrac{1}{3}, \tfrac{1}{5}\right) = \frac{2}{\frac{1}{\frac{1}{3}} + \frac{1}{\frac{1}{5}}} = \frac{2}{8} = \frac{1}{4}$$

We'll see: Can build an infinitely long bridge this way.

Q: Suppose given an infinite supply of blocks that are 2ft long and weigh 1 lb each



* mass is uniformly dist.

* How far can a stack of these blocks be made to extend from the edge of a table?