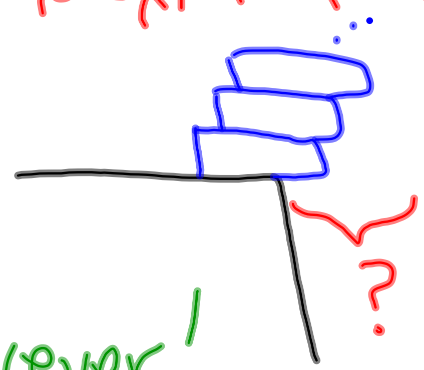
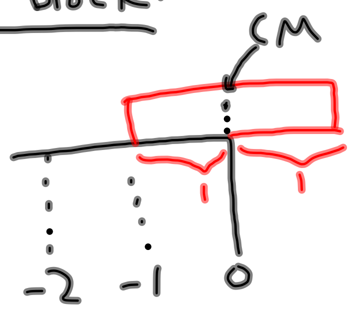
Q: Spse given an 'infinite supply of blocks, each 2 units long, with uniform mass of 1 kg.

How far can a stack of blocks be made to extend from the edge of a table:



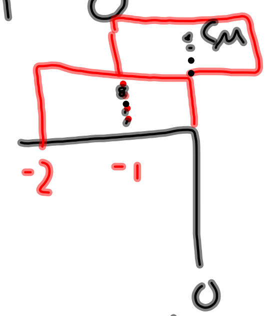
A: Forever!

1. <u>One block</u>:



Laws of physics dictate that CM <u>must</u> be over table.

2. <u>Two blocks</u>: Lift up first block & put a new block under it, with rightmost edge at 0



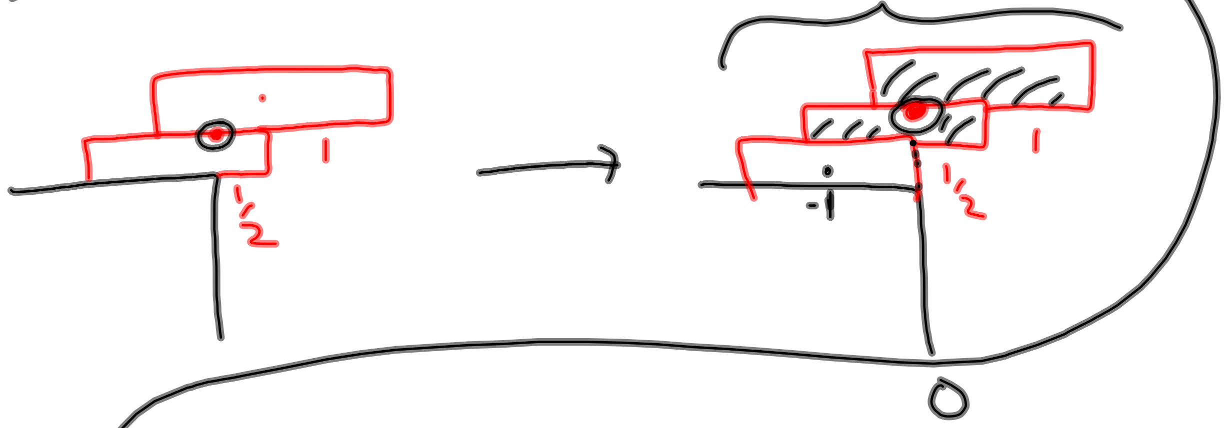
CM of the two blocks is at:

$$\frac{(-1)\cdot 1 + (0)\cdot 1}{2} = -\frac{1}{2}$$

$\Longrightarrow$ Can "shove" this 2-block stack $\frac{1}{2}$ unit to the right and it will still be standing.

3. Three blocks

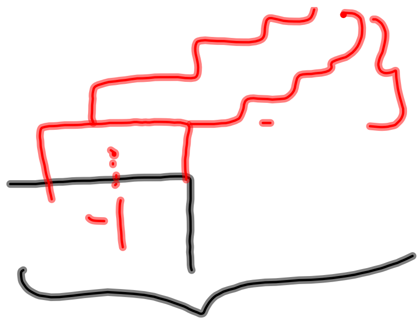
CM is



$$\frac{(-1)\cdot 1 + (0)\cdot 2}{3} = -\frac{1}{3}$$

$\Longrightarrow$ Can move the 3 blocks $\frac{1}{3}$ unit right

General



$\}$ $n-1$ blocks, CM @ 0

$$\text{new CM} = \frac{(-1)\cdot 1 + (0)\cdot(n-1)}{n} = -\frac{1}{n}$$

Thus: A stack of n-blocks can be made to reach

$$H(n) := 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} \text{ units}$$

from the table.

Claim: H(n) can be made arbitrarily large by making n very big:

$$\underbrace{1}_{\geq \frac{1}{2}} + \underbrace{\frac{1}{2} + \frac{1}{3}}_{> \frac{2}{4} = \frac{1}{2}} + \underbrace{\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}}_{> \frac{4}{8} = \frac{1}{2}} + \underbrace{\frac{1}{8} + \cdots + \frac{1}{15}}_{> \frac{8}{16} = \frac{1}{2}} + \cdots$$

Thus:

$$H(2^n - 1) > \underbrace{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots + \frac{1}{2}}_{n} = \frac{n}{2}$$

\# particles in universe

$\approx 10^{80} \approx$

$< 2^{320}$

$2^{2\,000\,000}$

Next: Functions: (Problems p.79 #6)

Q: Person A starts driving at 50 mph. Person B starts 3 hours later and tries to catch up. How long does it take if B travels at 75 mph?

A: Let t = time it takes to catch up.

$$\underbrace{50(3+t)}_{\text{dist covered by A}} = \underbrace{75t}_{\text{dist covered by B.}}$$

$\Rightarrow t = 6$ hours.

Further analysis:

Let $t :=$ time (hrs) after B starts driving

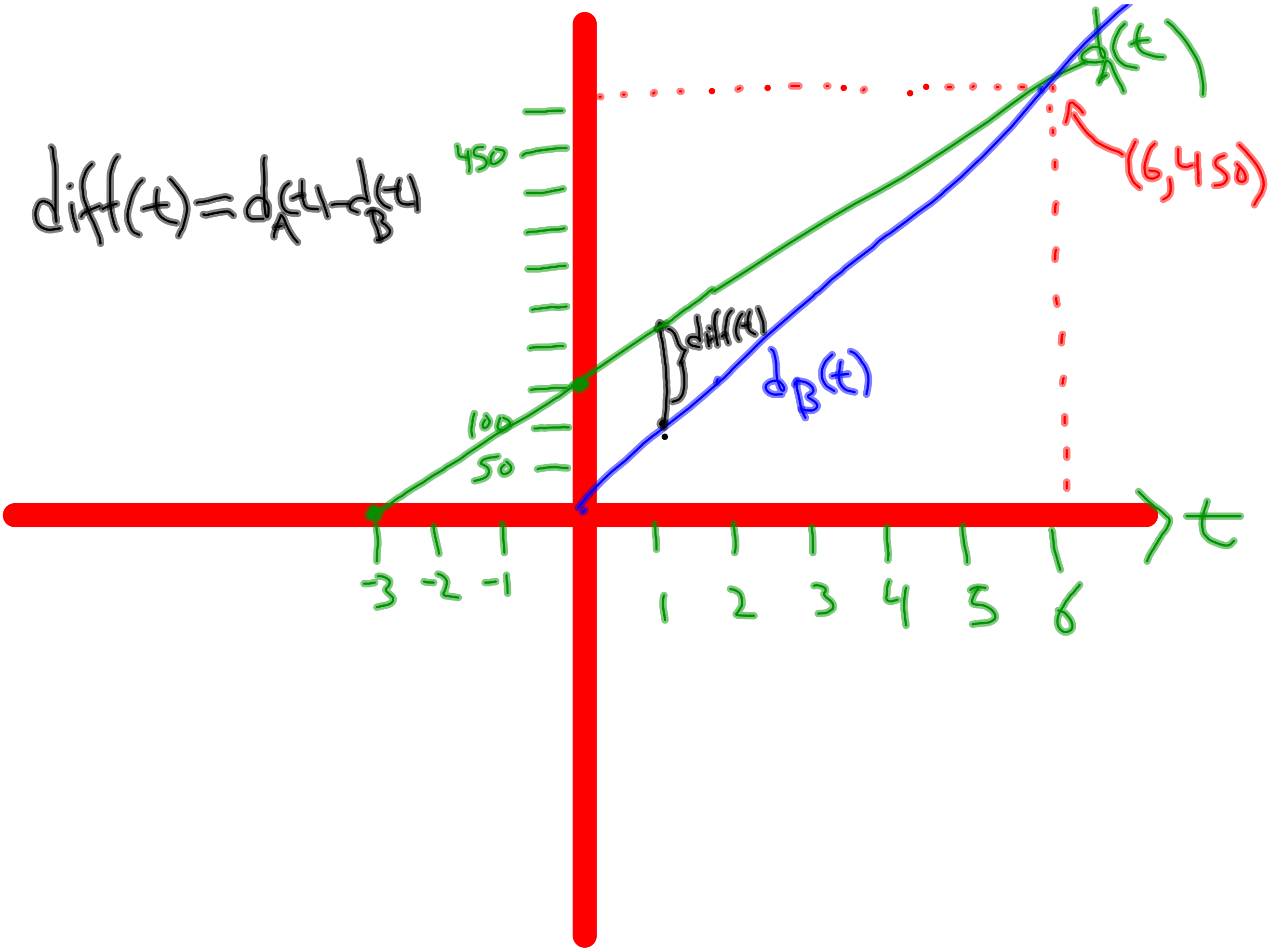
$d_A := d_A(t) :=$ dist. covered by A at time t

$d_B := d_B(t) :=$ __________ B _____ _____

$d_A(t) = 50(t+3)$

$d_B(t) = 75t$

$$diff(t) = d_A(t) - d_B(t)$$



Even further analysis:

* Generalize by having B travel at w mph
variable.

t = time needed to catch up

$$50(t+3) = w \cdot t$$

$$\Rightarrow \quad t = \frac{3}{\frac{w}{50}-1} = \frac{150}{w-50} \quad (w>50)$$

• Notice:

$$\frac{dt}{dw} = \frac{d}{dw}\left(\frac{3}{\frac{w}{50}-1}\right)$$

$$= -\frac{150}{(w-50)^2}$$

* The faster B goes, the less effect change in speed has on catch up-time.

In particular, $\left.\frac{dt}{dw}\right|_{w=75} = -\frac{150}{(25)^2}$

$$= -0.24$$

i.e. Instantaneous rate of change of t w.r.t. w, when $w=75$, is -0.24

$\Rightarrow$ If w is increased by a small amount Δw, then catch-up time is decreased by $\approx 0.24\Delta w$

General problem:

h = head start (hrs)

t = catch up time

v = speed of A (constant)

w = speed of B (constant)

$$t = \frac{h}{\frac{w}{v} - 1}$$