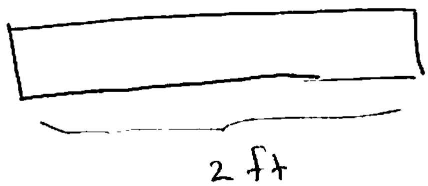
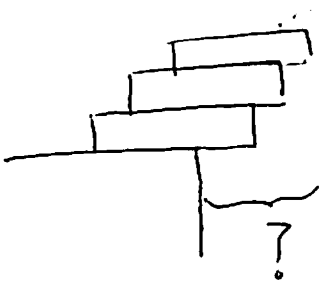
Q: Suppose given an infinite supply of blocks that are 2 ft long and weigh 1 lb. each.




Assume the mass is uniform.

~~Please stack these blocks~~

How far can a stack of these blocks be made to extend from the edge of a table?

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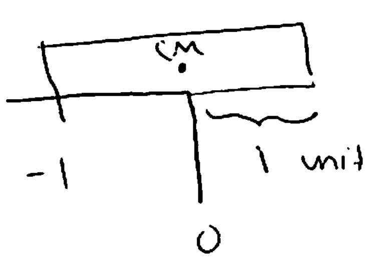
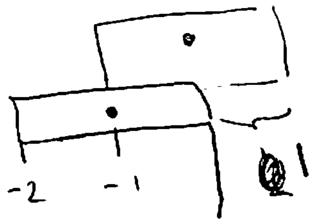
1 block: Laws of physics say that the Center of Mass (CM) must be over table



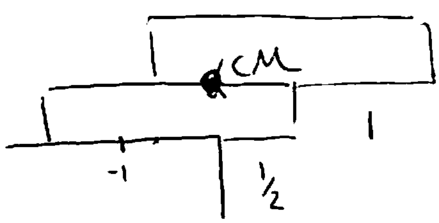

2 blocks: lift above block straight up, and place a block under it with right-edge at 0.



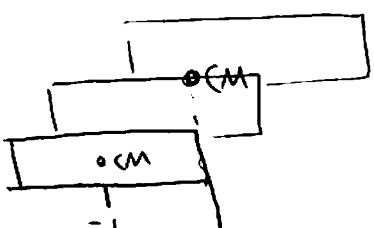

The CM of the <u>two</u> blocks is

$$\frac{(-1)\cdot 1 + (0)\cdot 1}{2} = -\tfrac{1}{2}$$

So we can push the two blocks $\frac{1}{2}$ unit to the right.




<u>3 blocks:</u> Lift, place block underneath




The new CM is at

$$\frac{(-1)\cdot 1 + 2\cdot(0)}{3} = -\frac{1}{3}$$

So can push it to the right $\frac{1}{3}$.

In general, CM of ~~2-block stack~~ n-block stack w/ bottom CM @ $-1$ and $(n-1)$ on top CM @ $0$ is $\frac{(-1)\cdot 1 + (0)(n-1)}{n} = -\frac{1}{n}$.

**Next:** Functions.

**Problems:** p. 79, # 6

**Q:** Person A starts driving at 50 mph.
3 hours later, person B tries to catch up.
How long does it take B to catch up
if they drive at 75 mph?

**Sol:** Let $t$ = time it takes B to catch up (hrs)

~~$d_A$ = distance traveled by A (miles)~~

~~$d_B$ = di~~

$$\text{Then}\quad \underbrace{50(\underbrace{t+3}_{\substack{\#\text{ hours}\\ \text{A has}\\ \text{been driving}}})}_{\substack{\text{distance A}\\ \text{has driven}}} = \overbrace{75\,\underbrace{t}_{\substack{\#\text{ hrs}\\ \text{B has}\\ \text{drive}}}}^{\text{dist B has driven}}$$

Solving for $t$ gives $t = 6$ hours.

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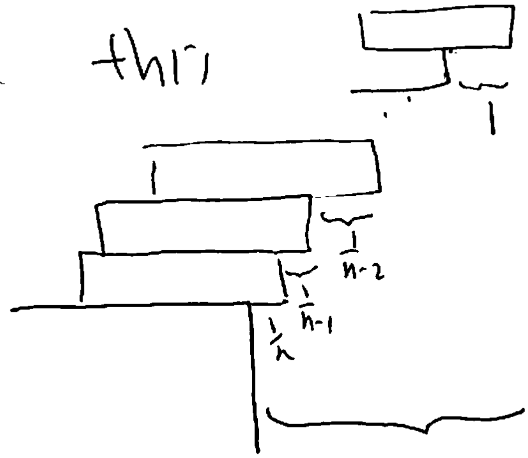
**Functions approach (I):** Let $t$ = time after B starts driving (hrs)

$d_A$ = distance traveled by A at time $t$

$d_B$ = dist traveled by B at time $t$.

Conclusion: We can make an $n$-block stack like this



$$1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}.$$

So how far can we get?

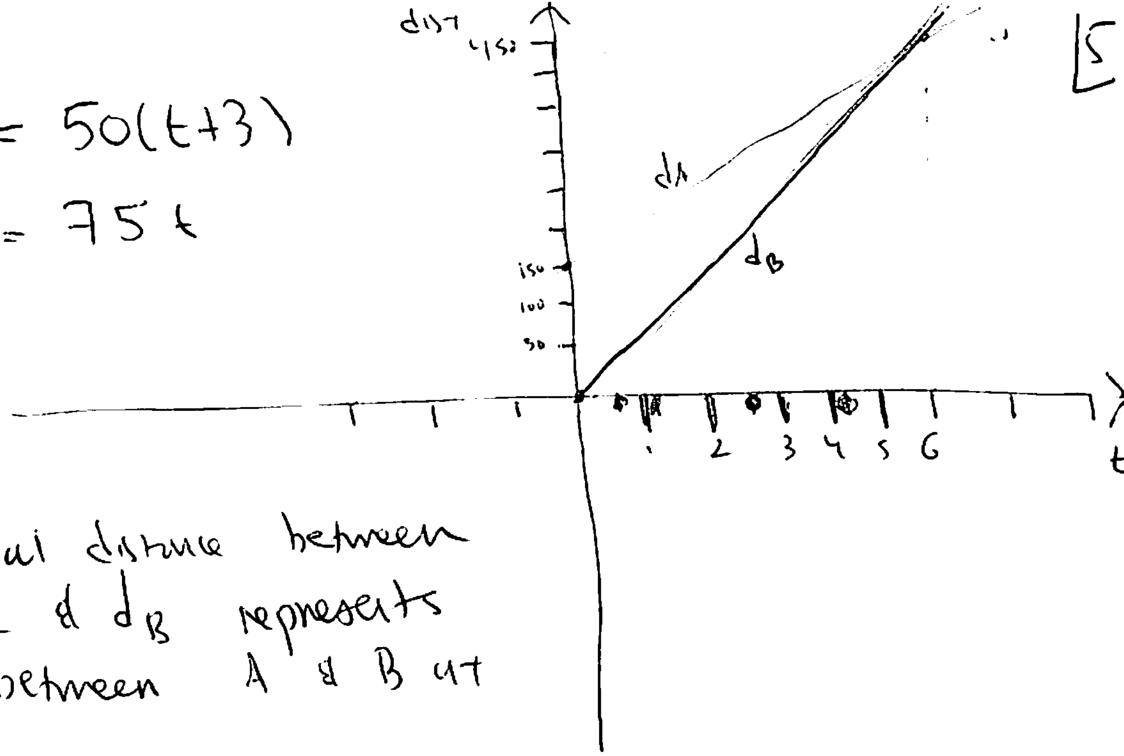
$$\underbrace{1}_{\geq \frac{1}{2}} + \underbrace{\frac{1}{2}+\frac{1}{3}}_{\geq 2\left(\frac{1}{4}\right)} + \underbrace{\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}}_{\geq 4\left(\frac{1}{8}\right)} + \underbrace{\frac{1}{8}+\cdots+\frac{1}{15}}_{\geq 8\left(\frac{1}{10}\right)} + \frac{1}{16}$$

$$\geq \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\cdots$$

Can make as **BIG** as we like!!

$$\left(\sum^{2^n-1} > n\cdot\frac{1}{2}\right)$$

<u>Then:</u> $d_A = 50(t+3)$
$d_B = 75t$




*NB: The vertical distance between the graphs of $d_A$ & $d_B$ represents the distance between A & B at time t.

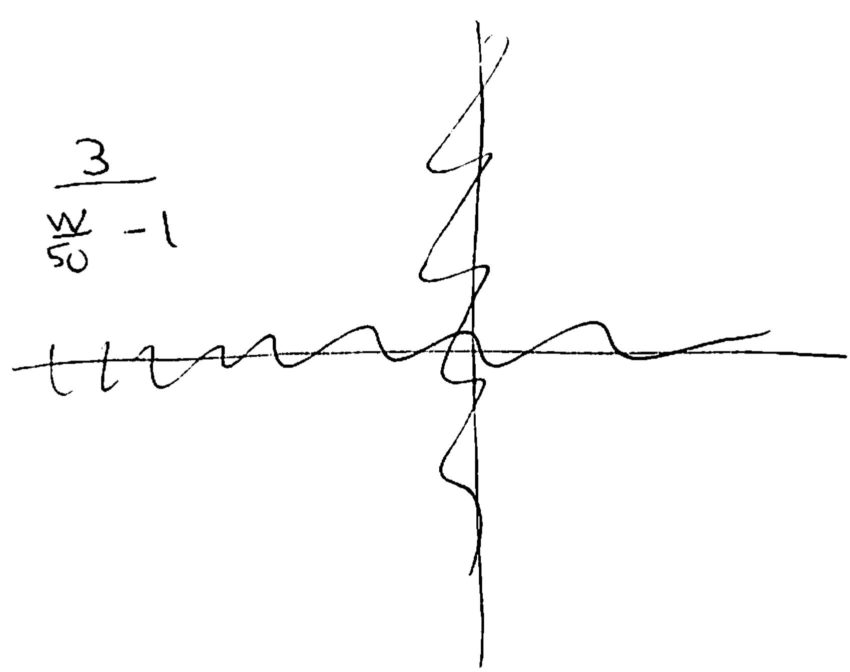
<u>Function Approach</u> (II: Generalization)

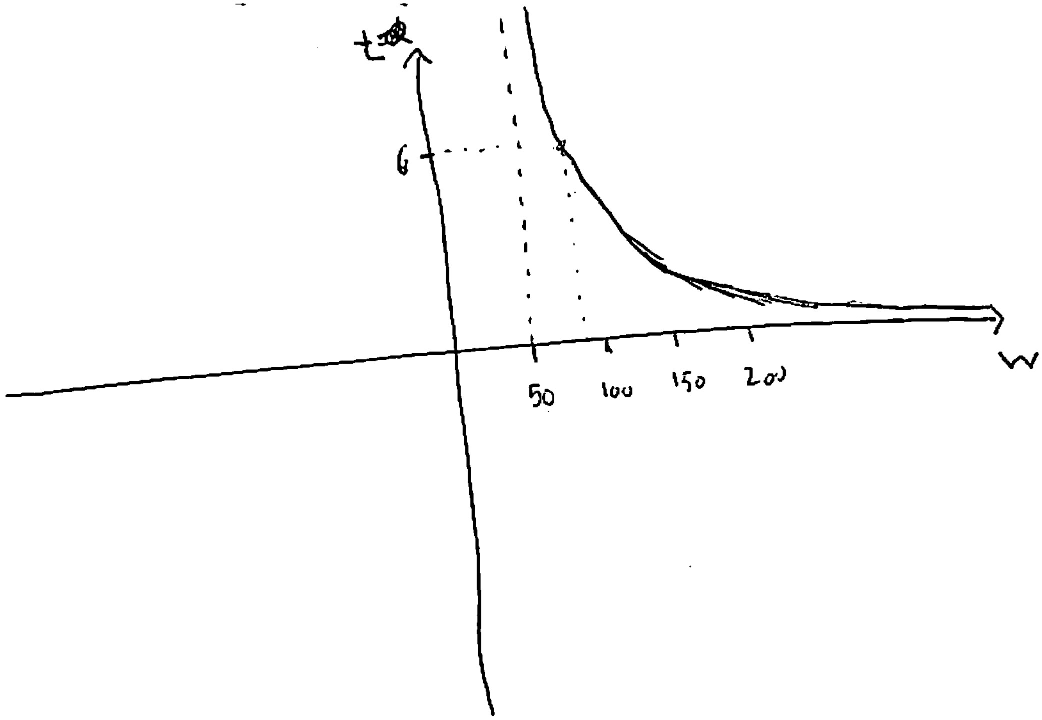
* We can generalize our problem by allowing the speed of B to <u>vary</u>:

Spse B travels at w mph. ~~Then~~ If t = # hrs it takes B to catch up, then

$$50(t+3) = wt$$

So $$t = \frac{150}{w-50} = \frac{3}{\frac{w}{50}-1}$$






Observations:

- As $w \to 50$, $t \to \infty$
- As $w \to \infty$, $t \to 0$
- Since $\frac{dt}{dw} = \frac{d}{dw}\left(\frac{150}{w-50}\right) = -\frac{150}{(w-50)^2}$

we see, for example, that at $w = 75$,

$\left.\frac{dt}{dw}\right|_{w=75} = \frac{-150}{625} = \frac{-30}{125} = \frac{-6}{25} = -0.24$ so a small increase $\Delta w$ in $w$ will decrease the catch-up time by $(0.24)\Delta w$.

- $\frac{dt}{dw} \to 0$ as $w$ goes to $\infty$.

What does this mean??

More generally: $t = \frac{h}{\frac{w}{v} - 1}$

$h$ = head start
$w$ = speed of B
$v$ = speed of A
$\left.\right\} v(t+h) = wt$