Problems: #3, p75

Q: What is a function?

* each input has unique out put
* vertical line test
* each input.... where?
* at most one output!
* each output can have multiple inputs.
* a relationship b/w variables
* a mathematical statement ...... st ....

* Def (modern): A function is a rule which assigns to each element of a set A exactly one element of a set B.

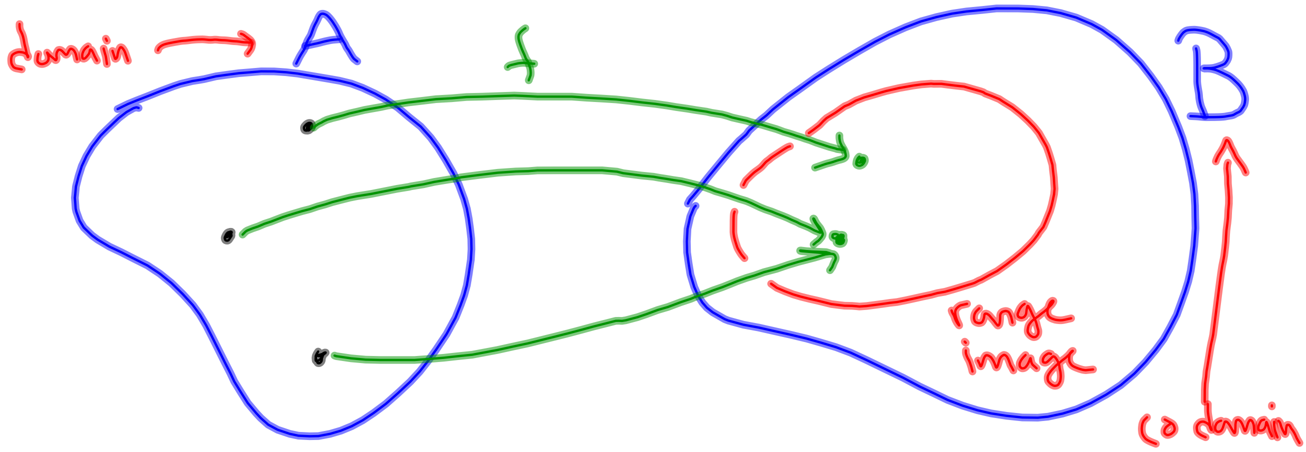
- will use "f" for function, and

$$f : A \longrightarrow B$$

domain (source) — A; codomain (target) — B

- image or range of f is the subset of B consisting of elts which are "outputs" of f.

- will write f(a) = the elt of B assigned to a by f.



- Sometimes we call a function a "map" or a "mapping".
  If $f(a)=b$, we say "f maps a to b"
  $f: a \mapsto b$
  or $a \mapsto b$
- f is <u>injective</u> or <u>one to one</u> (1-1) if $f(a)=f(a') \Rightarrow a=a'$
- f is <u>surjective</u> or <u>onto</u> if for every $b \in B$ there exists $a \in A$ s.t. $f(a)=b$ (range = co-domain)
- <u>bijective</u> = surjective + injective.

Examples:

- Functions can be specified by formulas
  - $f(x) \stackrel{\text{def}}{:=} x$
  - $f(x) \stackrel{\text{def}}{:=} \sqrt{x}$
  - $f(x) = \sin(e^{2^x})$
- Functions can be specified by descriptions:
  - $f(n) =$ the n^{th} prime number for $n = 1, 2, 3, \ldots$

    $f(1) = 2$ $f(3) = 5$

    $f(2) = 3$
  - $f(n) =$ the ave. yearly rainfall (in in) in Tucson, AZ in year $2000+n$ AD

    $n = 0, 1, 2, 3, \ldots$

- Can specify functions by tables:

x	$f(x)$
1	7
2	-3
3	24
4	1
5	24

$A = \{1,2,3,4,5\}$

$B = \mathbb{Z}$

$A =$ things I have tasted

$B = \{\text{yum}, \text{yuck}\}$

$a \in A$	$b = f(a) \in B$
apple	yum
orange	yum
bacon	yum
dirt	yuck

- Functions can be given by "piece-wise" defs

  - $f(x) = \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$

    $= |x|$

- Functions can be specified by a graph

encodes the function
f(a)=b iff (a,b)
lies on graph.

Def: Let $f: A \to B$ be a function. The graph of f is the subset

$$\Gamma(f) := \left\{ (a,b) \in A \times B \,\middle|\, \begin{matrix} a \in A \\ b = f(a) \end{matrix} \right\}$$

of

$A \times B$ = set of ordered pairs $(\underset{A}{\underset{\cap}{a}}, \underset{B}{\underset{\cap}{b}})$

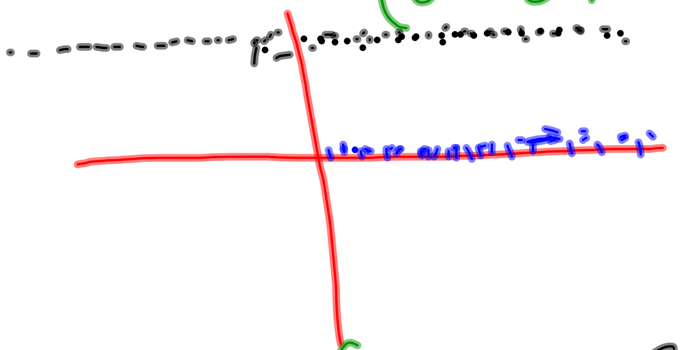
Notice The map $pr_A: A \times B \to A$
$(a, b) \mapsto a$

f a function $\Rightarrow$ $pr_A: \Gamma(f) \to A$ is bijective

Def (alternate): A function $f: A \to B$ is a subset $S \subseteq A \times B$ satisfying $pr_A: S \to A$ is bijective.

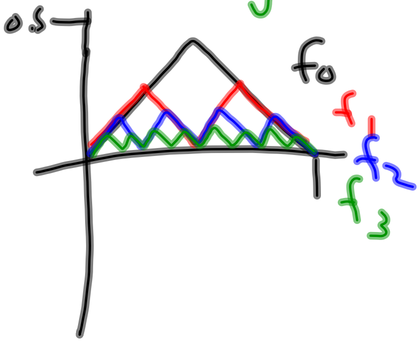
Some wild functions $f:\mathbb{R}\to\mathbb{R}$

- $f(x)=\begin{cases}1 & \text{if } x \text{ is rational}\\ 0 & \text{otherwise}\end{cases}$



nowhere continuous

- Takagi's function: $f_0(x)=\begin{cases} x & \text{if } 0\le x\le 1/2 \\ 1-x & \text{if } \frac{1}{2}<x\le 1\end{cases}$



$$f_k(x)=\frac{1}{2^k}f(2^k x)$$

$$f(x)=\sum_{k=0}^{\infty} f_k(x)$$