*Name*______________________

The number *e*

Work on this together in your group, but each person will turn in their work individually. Use your calculator and plenty of scratch paper. Write your answers on this paper—add your own paper if you need more space.

Consider the function $g(h) = (1+h)^{1/h}$.

1. Fill in the table for values of $g(h)$ for *h* near 0.

h	1	0.1	0.01	0.001	0.00001
$g(h)$					

2. To the nearest 0.000000001, estimate $\lim_{h\to 0} g(h) =$ ______________________.

This is an approximation of the irrational number ***e***. One definition is: ***e*** = $\lim_{h\to 0}(1+h)^{1/h}$. This ***e*** is not a function, it is just a number. So what's the big deal about ***e***? When we use the number ***e*** as the *base* of an exponential function, it has a very unique property. First, let's look at this numerically. Let $f(x) = e^x$. Write all your answers to ***nine*** decimal places.

3. What is the value of our function $f(x) = e^x$ at $x = 1$? $f(1) =$ ____________________.

Recall that the definition of the derivative function is given by $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. We will approximate the value of the derivative numerically at $x = 1$.

4. Fill in the table of values for smaller and smaller values of *h* (use 9 decimal places).

h	1	0.1	.001	.0001	.000000001
$\frac{f(1+h)-f(1)}{h}$					

5. Give your 9-decimal place approximation for $f'(1)$. $f'(1)$ = ___________________________

6. How does your answer compare to your answer for #3 above? Write an equation relating the value of the function at $x = 1$ to the value of the derivative at $x = 1$.

7. Now you get to make a choice. Pick any number you want and call it *p*. (Use a different number than those around you.) Write your value. $p =$ ___________

8. Repeat problems 3 through 6 with your value of *p*.

$f(p) =$ _____________________

Fill in the table of values for smaller and smaller values of *h* (use 9 decimal places).

h	1	.1	.001	.0001	.000000001
$\frac{f(p+h)-f(p)}{h}$					

Give your 9-decimal place approximation for the value of the derivative at *p*:

$f'(p) =$ _______________________

Write an equation relating the value of the function at $x = p$ to the value of the derivative at $x = p$.

9. Consult with your group and conjecture a guess as to the relationship between the value of the function $f(x) = e^x$ and the value of its derivative $f'(x)$ for all *x*.

We will now confirm your conjecture algebraically using the definition of the derivative. We want to show that for $f(x)=e^x$, its derivative $f'(x)$ is also equal to e^x. Follow the simplifications carefully:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0}\frac{e^{x+h}-e^x}{h} \\
&= \lim_{h\to 0}\frac{e^x e^h-e^x}{h} \\
&= \lim_{h\to 0}\frac{e^x(e^h-1)}{h} \\
&= e^x\cdot\lim_{h\to 0}\frac{(e^h-1)}{h}
\end{aligned}$$

All we have left to find is the value of $\lim_{h\to 0}\frac{e^h-1}{h}$ .This limit can be found using our definitions, plus a bit of algebra. Since ***e*** = $\lim_{h\to 0}(1+h)^{1/h}$, we can substitute this expression into our limit.

10. Show that $\lim_{h\to 0}\frac{e^h-1}{h}=1$.

With this limit solved, we now have $f'(x)=\frac{d}{dx}\left(e^x\right)=e^x$

.

Remember that ***e*** is not a function—it is just a number. But when it is used as the base of an exponential function, the resulting function $f(x)=e^x$ has a derivative that is identical to itself for all *x*. In other words, at every point of the graph of $f(x)=e^x$, the slope at that point matches the value of the function itself at that point. $f(x)=e^x$ is the only function in mathematics for which this is true at every point.

That's why *e* is the *natural exponential base.*