MATH 124 - Section 23, Fall 2007
Exam 1a

NAME (print): ____

DIRECTIONS:

1. For multiple choice, CLEARLY mark your choice by a circle. If I cannot determine what you have circled, the question will be marked wrong.
2. For the long answer problems, include all steps. A correct answer will not necessarily result in full credit, unless you show each step.

Signature: ____

The first 8 questions are multiple choice and are worth 7 points each.

1. Suppose $g(x)$ is a function with $g(10) = 7$ and $g'(10) = -2$. Estimate $g(12)$ using a tangent line approximation.

   (a) 9, (b) 3, (c) 5, (d) 11, (e) 12.

   Answer: (b) Write the equation for the line tangent to g at $x = 10$. Since $g'(10) = -2$ we know the slope of this line is $m = -2$. Since $g(10) = 7$ we know the point $(10, 7)$ is on this line. Given a point and slope, we can write the equation of the line as

$$y - 7 = -2(x - 10) \;\; \rightarrow \;\; y = -2x + 27.$$

   We estimate $g(12)$ by using the value of the tangent line at $x = 12$. Thus

$$g(12) \approx -2(12) + 27 = -24 + 27 = 3.$$

2. Evaluate the following limit

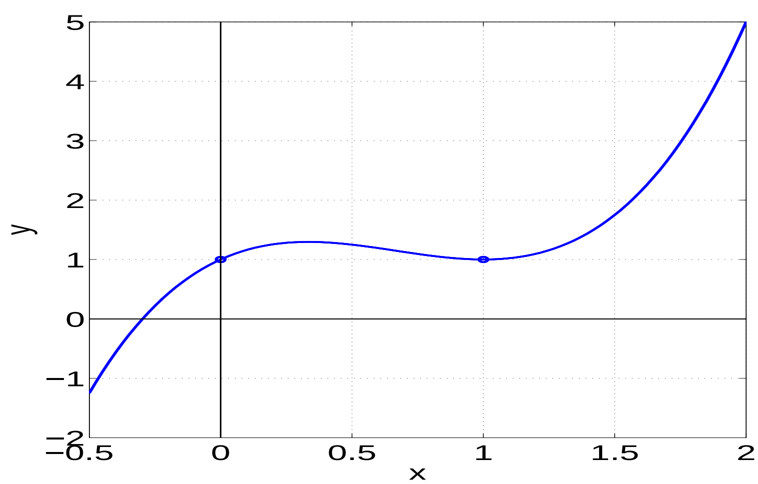
$$\lim_{x \to 2^-} \frac{x^2 - x - 2}{|x - 2|}.$$

   (a) 3, (b) ∞, (c) -3, (d) 1, (e) Does Not Exist.

   Answer: (c) We are evaluating the limit as x approaches 2 from the left, so the values of x will all be less than 2. In this case, $x - 2$ is always negative. Thus $|x - 2| = -(x - 2)$. Factoring the top, we have $x^2 - x - 2 = (x - 2)(x + 1)$. Putting these two simplifications in, we get

$$\lim_{x \to 2^-} \frac{x^2 - x - 2}{|x - 2|} = \lim_{x \to 2^-} \frac{(x - 2)(x + 1)}{-(x - 2)} = -\lim_{x \to 2^-} (x + 1) = -3.$$

3. Find a possible equation for the following graph.



   (a) $y = 2\,(x - 1)\,x$, (b) $y = (x - 1)^2\,x^2$, (c) $y = 2\,(x - 1)\,x - 1$, (d) $y = 2\,(x - 1)$, (e) $y = 2\,(x - 1)^2\,x + 1$.

   Answer: (e) First, we note the graph must be odd degree, since it begins and ends in different directions. Thus we can eliminate (a),(b), and (c) as choices. Clearly, this is not a line, so we can eliminate (d). Thus, the answer is (e). We can double check this answer by looking at a few points on the graph. If $y = 2\,(x - 1)^2\,x + 1$ then we have $x = 0$,$y = 1$ and $x = 1$, $y = 1$ as point on our graph. Sure enough, these are the two points that are marked.

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4. Find the average rate of change of $f(x) = x^3 - 1$ between $x = -2$ and $x = 1$.

(a) 0, (b) -2, (c) 2, (d) 1, (e) 3.

Answer: (e) Average rate of change over an interval is simply the change in y over the change in x for that interval. So we want:

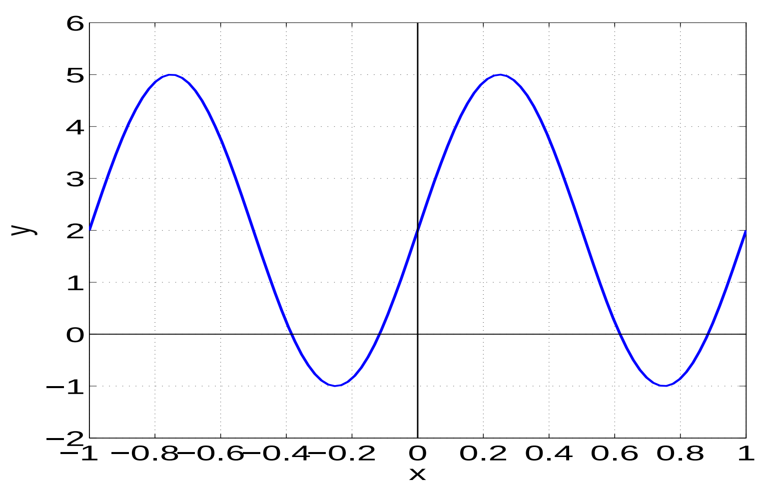
$$\frac{\text{change in } y \text{ over } x = -2 \text{ to } x = 1}{\text{change in } x \text{ over } x = -2 \text{ to } x = 1} = \frac{f(-2) - f(1)}{(-2) - 1} = \frac{-9 - 0}{-3} = 3.$$

5. Which of the following is the derivative of $f(x) = 6x^e$

(a) $6ex^e$, (b) e^{6x}, (c) $6x^e$, (d) $6e$, (e) $6ex^{e-1}$.

Answer: (e) This is simply a multiple of a power function. So think of it as $f(x) = 6x^n$. It does not matter than n is irrational in this case, it only matters that n is a constant. Thus $f'(x) = 6nx^{n-1}$ or $6ex^{e-1}$in our case.

6. What is the amplitude of the following periodic function?



(a) 2, (b) 3, (c) 2π, (d) π, (e) 6.

Answer: (b) The amplitude of a periodic function is defined at one half of the span. This is one half of the max value minus the min value. The maximum of our function is 5 and the minimum is -1. So we get

$$A = \frac{1}{2}(Max - Min) = \frac{1}{2}(5 - (-1)) = \frac{6}{2} = 3.$$

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The rest of the problems are all workout. Please show ALL of your work for full credit.

7. Consider $f(x) = \dfrac{2x^2 - 18x + 36}{x^2 - 8x + 15}$

(a) [6pts] Find all vertical asymptotes.

Answer: Vertical asymptotes are where the limit of the function approaches positive or negative infinity. For a problem like this (a rational function) this will typically be where the function is undefined. We must first factor the function, to make sure no terms cancel. We factor the top as $2x^2 - 18x + 36 = 2(x^2 - 9x + 18) = 2(x-6)(x-3)$. We factor the bottom as $x^2 - 8x + 15 = (x-3)(x-5)$. Rewriting, we have

$$f(x) = \frac{2x^2 - 18x + 36}{x^2 - 8x + 15} = \frac{2(x-6)(x-3)}{(x-5)(x-3)}.$$

We can cancel the $(x-3)$ from the top and bottom and get

$$f(x) = \frac{2x^2 - 18x + 36}{x^2 - 8x + 15} = \frac{2(x-6)}{(x-5)}.$$

We note this is undefined at only $x = 5$. So this is the only possible vertical asymptote. Taking a limit from each direction, we find

$$\lim_{x \to 5^-} \frac{2(x-6)}{(x-5)} = \infty \text{ and } \lim_{x \to 5^+} \frac{2(x-6)}{(x-5)} = -\infty$$

so $x = 5$ is the vertical asymptote.

(b) [6pts] Find all horizonal asymptotes.

Answer: Horizonal asymptotes are found simply by taking the limit of $f(x)$ as $x \to \pm\infty$. Since the top is a polynomial, and the bottom is a polynomial, when we take the limit to $\pm\infty$ we can ignore the higher order terms in each. Thus

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{2x^2 - 18x + 36}{x^2 - 8x + 15} = \lim_{x \to \pm\infty} \frac{2x^2}{x^2} = \lim_{x \to \pm\infty} 2 = 2.$$

Thus $y = 2$ is the only horizontal asymptotes.

(c) [6pts] Find all zeros.

Answer: The zeros of a rational function are simply where the numerator is equal to zero. Since we have already factored and canceled, we want to solve

$$f(x) = \frac{2x^2 - 18x + 36}{x^2 - 8x + 15} = \frac{2(x-6)}{(x-5)} = 0.$$

This is where $2(x-6) = 0$, which is clearly at $x = 6$.

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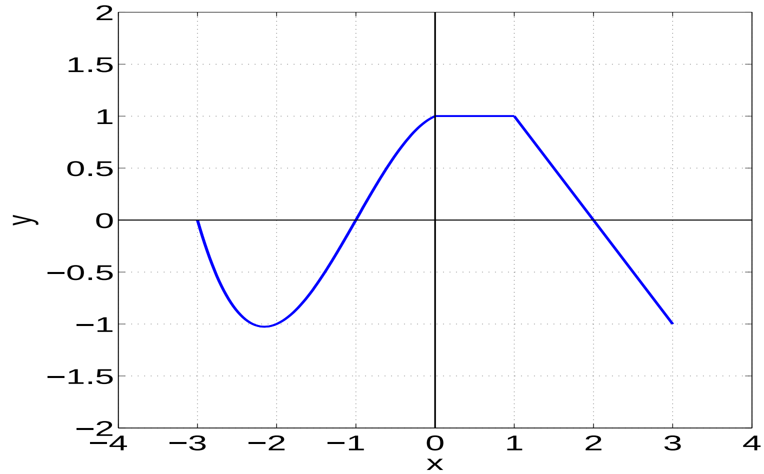
8. Let $h(x)$be represented by the following graph:



(a) [5pts] What is $h(-1)$?

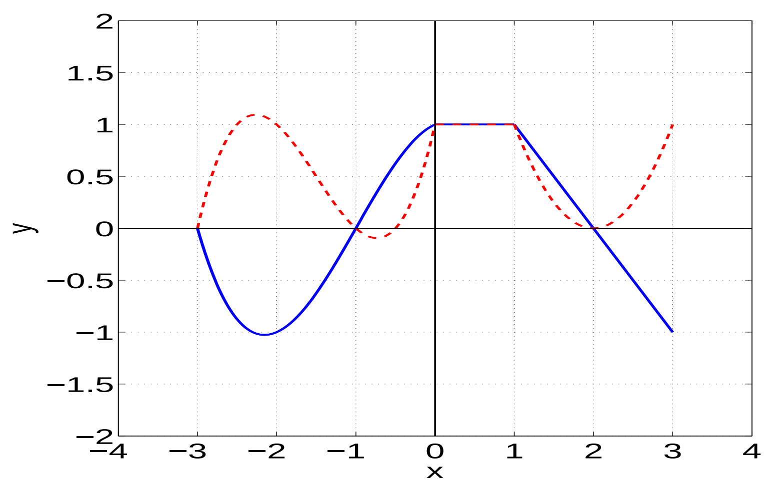
Answer: You are supposed to simply find the value of h at $x = -1$ (that's what $h(-1)$ means!). Looking at the plot, at $x = -1$ we have $h(-1) = 0$.

(b) [5pts] What is $(h(3))^2$?

Answer: You want to find $h(3)$ first. The value of the graph at $x = 3$ is $h(3) = -1$. Then $(h(3))^2 = (-1)^2 = 1$.

(c) [6pts] Sketch $(h(x))^2$.

Answer: We square h. The easiest way to graph this is to take individual values of $h(x)$ and just square them. If $h(x) = 0$, then $(h(x))^2 = 0$. If $h(x) = 1$ then $(h(x))^2 = 1$. Similarly if $h(x) = -1$ then $(h(x))^2 = 1$. If you plot all of these points (where $h(x) = 0, 1$ or -1) correctly, then you should get a plot similar to the following (where $(h(x))^2$ is the dotted curve):



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9. The quantity, Q, of radioactive carbon-14 remaining t years after an organism dies is given by $Q = Q_0 e^{-.00012t}$ where Q_0 is the initial quantity.

(a) [5pts] A bone uncovered at an archaeological dig has 40% of its original amount of carbon-14 present. How old is the bone? (Round to the nearest year)

Answer: Use the given formula for Q. We are given that a bone has 40% of its original amount. If we assume it starts with 100%, then $Q_0 = 100$ and $Q = 40$ (since there is 40 percent left). We then have $40 = 100e^{-.00012t}$ which simplifies to $0.4 = e^{-.00012t}$. We take the ln and get

$$\ln 0.4 = -.00012t \text{ or } t = \frac{\ln 0.4}{-.00012} \approx 7636\text{yrs}.$$

(b) [5pts] What is the half-life of carbon-14?

Answer: We do almost the exact same thing as part (b), except we want to know when there was 50% left. Following the same steps, we get

$$\ln 0.5 = -.00012t \text{ or } t = \frac{\ln 0.5}{-.00012} \approx 5776\text{yrs}.$$

This is the end of Exam I