MATH 124 - Section 23, Fall 2007
Practice Exam 4

1. (Section 5.1) When performing an experiment, one finds the velocities of a particle is given by:

t	0	1	1.5	2
$v(t)$	1	2	0	3

Give upper and lower estimates for the distance the particle traveled from $t = 0$ to $t = 2$.

2. (Section 5.1) Consider $f(x) = x^2$ on the interval $[0, 2]$. What is the value of the Riemann sum based on left-hand end-points using 4 equal subintervals? What is Δt? Is your answer higher or lower than $\int_0^2 x^2\,dx$?

3. (Section 5.2) Without using a calculator, find $\int_{-1}^{1} x + 1\,dx$. (NOTE: Be able to find the area under a given graph, ie, add and subtract areas of triangles and rectangles).

4. (Section 5.2) Find $\int_0^{\sqrt{\pi}/2} t\sec^2\left(t^2\right)\,dt$.

5. (Section 5.3) The population of Mexico can be modeled by

$$P(t) = 67.38\,(1.026)^t\,,$$

where P is in millions of people and t is in years since 1980. Use this to predict the average population of Mexico between the years 2000 and 2020.

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6. (Section 5.3) A turkey at $70°F$ is put into a $350°F$ oven at $t = 0$. The temperature of the turkey, $T(t)$, is changing at a rate given by $T'(t) = 20\,(1.1)^{t\,\circ}\,F$ per hour, where t is in hours. Estimate, to the nearest degree, the turkey's temperature after 4 hours.

7. (Section 5.4) Suppose we have $f(x)$ such that $\displaystyle\int_1^3 f(x)\,dx = 7$, $\displaystyle\int_0^2 f(x)\,dx = 3$, and $\displaystyle\int_2^3 f(x)\,dx = 13$. Find $\displaystyle\int_0^1 f(x)\,dx$.

8. (Section 5.4) Calculate the area of the region bounded by $y = 2x^2 - 1$ and $y = x$.

9. (Section 6.1) Use a graph of $f(x) = 2\sin(x^2)$ to determine where an antiderivative, F, of this function reaches its maximum on $0 \le x \le 3$. If $F(1) = 5$, find the maximum value attained by F.

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10. (Section 6.2) Evaluate each indefinite integral (in other words, determine the most general antiderivative of each integrand).

(a) $\displaystyle\int \frac{2}{\sqrt{1-x^2}}\,dx$

(a) $\displaystyle\int \left(\frac{1+z^2}{z}\right)\,dz$

(a) $\displaystyle\int \frac{\sin(x+1)\sec(x+1)}{\cos(x+1)}\,dx$

(a) $\displaystyle\int ae^{5t}\,dt$

11. (Section 6.2) Find the exact positive value of c if the area between the graph of $f(x) = x^2 - c^2$ and the x-axis is $\dfrac{256}{3}$.

12. (Section 6.2) If $f''(x) = 3x - 1$, $f'(0) = 2$, $f(0) = 6$ then what is $f(1)$?

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13. (Section 6.3) Solve the following differential equation with the specified conditions

$$\frac{dy}{dx} = \frac{8}{\pi^2}x + \cos x, \; y\left(\frac{\pi}{2}\right) = 3.$$

14. (Section 6.3) After a meltdown at a nuclear power plant a large amount of plutonium is spilled. The plutonium naturally decays at a rate of approximately 5% a month. Additionally, a cleanup crew from the Department of Energy cleans a small portion of the power plant once a month, eliminating approximately 100 grams of plutonium each time. Find a differential equation for the amount of plutonium, P, in grams, left after t months.

15. (Section 6.4) If $F'(t) = \sin t \cos t$ and $F(0) = 1$ then what is $F(\pi)$?

16. (Section 6.4) Consider $f(x) = \int_0^{x^2} \sin t \, dt$. Find $f'(1)$.

17. (Section 6.4) Let $g(x) = \displaystyle\int_4^{\sqrt{x+6}} \sqrt{t^4+9}\, dt$ for $x > -6$. Compute $g'(x)$.

This is the end of Example Exam 4