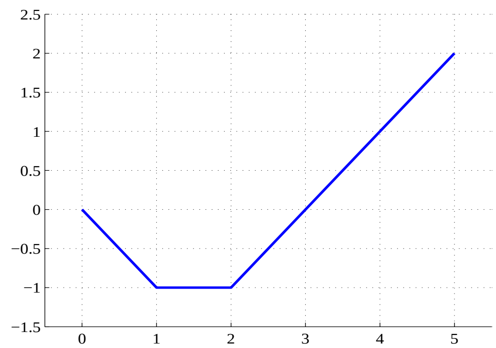
MATH 124 - Section 23, Fall 2007
Sample Final Exam Questions

This is a collection of questions which you may use for extra help reviewing for the final. This is not meant to cover all topics on the final. I have not completed every question on this sample exam, so there may be some errors or some questions that are extremely hard. These questions are in no particular order.

Please try to do the questions without your calculator (even the definite integrals).

1. If $f(x) = x^4 + 1$ and $g(x) = \sqrt{x+1}$ then what is $f(g(3))$?

2. Find $\int_0^4 f(x)\,dx$ if $f(x)$ is given by



3. What are the inflection points of $f(x) = e^{-x^2+1}$?

4. Find the area between $y = x^2 - 2x$ and $y = 12 - x$.

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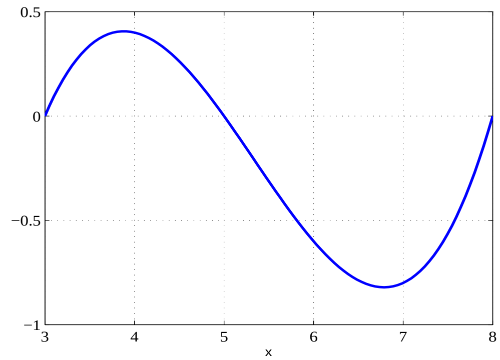
5. If the following graph is of $f''(x)$ then where is $f'(x)$ increasing?



6. Find the x value where the global (absolute) minimum of $y = (x+2)^2(x-3)$ is attained if $-3 \leq x \leq 3$?

7. If $f(x) = \ln\left(x^2 + e^x\right)$ then what is $f'(13)$?

8. The half-life of strontium-90 is 29 years. In 1964 strontium-90 was released during nuclear weapons testing and was absorbed into people's bones. How many years until 20% of the original amount absorbed remains? (Round to the nearest year)

9. If $h(x) = f(g(x))$then using the following information evaluate $h'(1)$.

$$f(1) = 1,\ f(2) = 3,\ f(7) = 6,\ f(15) = 2,\ f'(2) = 9,\ f'(1) = 1$$

$$g(1) = 2,\ g(2) = 6,\ g'(1) = 7,\ g'(2) = 15.$$

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10. The rate at which a population of rabbits on an island is changing is given by $r(t) = 50\left(e^{-0.2t} - e^{-0.3t}\right)$ where t is in months. If there are 2 rabbits on the island originally, then how many rabbits are there after 6 months? (Round to the nearest integer).

11. A product costs \$100 today. If the price decreases by 5% a day, then how much does the product cost a week from today? (Round to the nearest dollar)

12. The velocity of a car (in miles per hour) is given by $v(t) = \sin(2t) + \frac{t}{2}$ where t is in hours. If the car starts from home, then how far away from home has it traveled in 2 hours?

13. If $g(x) = \cos 2x$, then what is $g^{(5)}(x)$?

14. What is the period of $y = 2\sin\left(\frac{\pi}{4}t\right) + \pi$?

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15. Find the equation of the line passing through the points $(2, 7)$ and $(5, 3)$.

16. Find $\lim_{x \to 0} \frac{4x}{\sin(2x)}$.

17. Find $\int_0^1 \frac{dx}{1 + x^2}$.

18. Find the linear approximation of $f(x) = (1 + x)^{\frac{1}{3}}$ near $x = 0$.

19. Find $\lim_{x \to 0} (1 + x)^{1/x}$.

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20. Find $g(x)$ so that $g'(x) = \frac{1}{2x \ln x}$.

21. Find the equation of the tangent line to $y^5 + 3x^2y^2 - x^3 = 43$ at $(1, 2)$.

22. Find $\lim_{x \to 9^-} \frac{x+9}{x^2(x-9)}$.

23. Find the local min/max for $f(x) = |6x - x^2|$.

24. Find $\int_4^9 \sqrt{x} - x^{-3/2}\, dx$.

25. Find $\int_0^1 \frac{e^{3x}}{3 + e^{3x}}\, dx$.

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26. Find the distance between the point $(0, 1)$ and the line $x + 2y = 1$.

27. What is the area of the largest rectangle that can be inscribed in a triangle with vertices $(0, 0)$, $(0, 2)$ and $(1, 0)$ if two edges of the rectangle are along the axes?

28. The Texas Parks and Wildlife Department decides to stock a lake with approximately 1000 large mouth bass. The bass reproduce which results in a 50% increase of bass each year. Find a differential equation which, when solved, gives the number of bass in the lake in a given year.

29. Find $\lim_{x \to -\infty} xe^x$.

30. A snowball melts so that its surface area decreases at a rate of 2 cm^2/min, find the rate at which the radius decreases when the radius is 8cm.

31. Two cars start moving from the same point. One travels south at 40mi/h and the other travels west at 45mi/h. At what rate is the distance between the cars increasing two hours later?

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32. Evaluate each of the following

(a) $\dfrac{d}{dx} \arctan\left(\dfrac{x^2-1}{\ln(x)}\right)$

(b) $\dfrac{d}{dz}\left(e^{2z}+1\right)^3$

(c) $\dfrac{d}{dt} a e^{bt}\sin(ct)$

(d) $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

(e) $\displaystyle\int \frac{1}{\cos 2t\cdot\sin 2t}\,dt$

(f) $\displaystyle\int x^2+\sin x\,dx$

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33. The horizontal asymptote of the curve $\dfrac{x^2+2x+1}{2x^2-1}$ is given by what equation?

34. Given $f(x) = \dfrac{1}{x^2}$ calculate $f'(x)$ using ONLY the definition of the derivative (in other words, you may not use any differentiation rules).

35. Find parametric equations of the line tangent to the curve

$$\begin{aligned} x &= t^3+t+1 \\ y &= t^2-1 \end{aligned}$$

at the point $(3,0)$.

36. Consider the function $f(x) = \left\{ \begin{array}{ll} 1+cx^2, & x<1, \\ x^2+cx, & x \geq 1. \end{array} \right\}$

(a) Show that $f(x)$ is continuous at $x=1$.

(b) Find c so that $f(x)$ is differentiable at $x=1$.

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37. For $f(x) = \dfrac{2x-3}{2x+1}$, calculate $f^{-1}(x)$.

38. Calculate $\lim_{x \to 0} \dfrac{e^{\sin x} - 1}{x}$.

39. Find the equation to the line tangent to the curve $x^4 + y^2 = 1$ at the point $(1, 1)$.

40. A ball follows a trajectory given in parametric form by $x(t) = \cos(t) + \sin(t)$, $y(t) = \cos(t) + 1$. What is the slope of the tangent line at the point $(1, 2)$?

41. Let Q be a polynomial of degree 2 such that $Q(1) = 2$, $Q'(0) = 1$ and $Q''(0) = 3$. What is $Q(2)$?

42. Suppose g is the inverse function of f. If $f(1) = 2$,$f'(2) = 3$, and $f'(1) = 3$ then what is $g'(2)$?

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43. A 10 foot ladder is leaning against a wall. if the foot of the ladder is sliding away from the wall at a constant rate of 1foot per minute, how fast is the top of the ladder sliding down the wall when the foot of the ladder is 4 feet from the wall?

44. Find the 2^{nd} derivative of $f(x) = \sqrt{x+1}$.

45. Fencing is required to enclose a rectangular field with 1000 square feet of area. The north side requires fencing which costs $5 per foot, while the other sides require fencing costing $3 per foot. How long must the north side be in order to minimize the cost?

46. A car is accelerating along a straight road. The acceleration after t seconds is given by

$$a(t) = 30t + 8\,\mathrm{ft}/\mathrm{sec}^2.$$

(a) Find the velocity $v(t)$ if $v(0) = 50\mathrm{ft}/\mathrm{sec}$.

(b) Find the position $x(t)$ is the initial position is 100 feet down the road.

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47. Evaluate $\lim_{x \to 2^+} \frac{|3x-6|}{6-3x}$.

48. Find parametric equations for the line passing through the points $(1,1)$ and $(2,3)$.

49. Let $L(x)$ be the linear approximation of $f(x) = \sqrt{x}$ at $a = 10$. Compute $L(9)$.

50. Consider the curve C given by the parametric equations

$$x(t) = te^t, \; y(t) = \sqrt{t+1}, \; -\infty < t < \infty$$

(a) Find dx/dt and dy/dt.

(b) Find dy/dx in terms of the parameter t .

(c) Obtain an equation of the tangent line (in terms of x and y, NOT t) to C at the point $(0,1)$.

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51. What is the absolute (global) maximum of $f(x) = \dfrac{1}{x^2 - 6x + 10}$ on the interval $[0, 4]$.

52. Assume that during the first three minutes after a foreign substance is introduced into the blood, the rate at which new antibodies are produced (in thousands of antibodies per minute) is given by

$$r(t) = \frac{t}{t^2 + 1}$$

where t is in minutes. Find the total quantity of new antibodies in the blood at the end of three minutes (round to the nearest whole number of antibodies).

53. A city as an annual population growth rate of 2%. Assuming exponential growth (continuous growth), how many years will it take the population to triple?

54. An animal skull has 25% of the carbon-14 that was present when the animal died. If the half-life of carbon-14 is 5730 years, find the approximate age of the skull (to the nearest year).

55. The depth of water in a tank oscillates once every 6 hours. If the smallest depth is 1 ft and the largest depth is 5 ft, find a formula for the depth in terms of time, measured in hours. Assume the water level starts at 3ft.

This is the end of the sample questions