

Letter to the Editor concerning Griffiths-Omnès article

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The article by Robert Griffiths and Roland Omnès is an attempt to provide an interpretation of quantum mechanics that eliminates the concept of measurement. It provides excellent reasons for getting rid of measurement. However, it also raises troubling questions.

As Griffiths and Omnès emphasize, the relation between probability and quantum mechanics is subtle. In the mathematical theory of probability, there is a given family of events. Each event has a probability. The probabilities satisfy the additivity property: For every pair of events A, B , the probability of the event A is the sum of the probability of the event (A and B) with the probability of the event (A and not B). The additivity property is necessary if the probabilities are to have a frequency interpretation.

There is a somewhat analogous structure in quantum mechanics, and it is natural to define a quantum event to be a projection operator. The quantum state assigns a probability to each quantum event. Commuting projection operators are compatible quantum events. The conjunction (A and B) of compatible quantum events is the operator product AB . The identity operator I that projects onto the entire Hilbert space corresponds to an event that is sure to happen. The negation (not B) is then $I - B$. For each family of compatible quantum events, the probabilities of the events in the family satisfy the additivity property. The consistent-histories theory deals with families of quantum events that need not be compatible. If the events in such a family satisfy a consistency condition relative to the quantum state, then again their probabilities obey the additivity property. Every compatible family of quantum events is a consistent-history family.

Suppose (as is usual in physics) that the physical meaning of probability is given by the frequency interpretation. As a precaution, however, consider that this interpretation may be relative to the consistent-history family. That is, given a consistent-history family, for each quantum event in the family there is a corresponding physical event to which the frequency interpretation applies.

In this spirit, consider the following premise concerning a system undergoing a certain physical process: The probabilities for each consistent-history family describe the frequencies at which physical events occur when the physical process occurs repeatedly. This premise is denied by an interpretation of quantum mechanics in which there are corresponding physical events only when a measurement is being performed on the system. In such interpretations, only the probabilities for quantum events in the consistent-history family selected for measurement describe the frequencies at which physical events occur. The premise could also be denied by an interpretation of quantum mechanics in which there are corresponding physical events only when a physicist chooses to reason about them. That would be selection by whim, rather than by measurement. However, it should hold for any interpretation in which there is no particular context that gives preference to one consistent-history family over another. The following argument shows that, under one additional and rather natural assumption, this premise leads to a contradiction.

The assumption links different compatible families: If one compatible family is contained in another compatible family, then the physical events in the smaller family occur precisely when the corresponding physical events in the larger family occur. One can imagine a theory in which this assumption is violated. For example, consider a system of two distinguishable spin $1/2$ particles. Say B is a quantum event associated with the first particle (a certain spin component is $1/2$), while C' is a quantum event associated with the other particle (some other spin component is $-1/2$). One can consider these spin components of the two particles together. Then the compatible family is generated by B, C' . Or one can single out the first particle and ignore the other. Then the compatible family is generated by B alone. It might happen in a particular realization that the physical event defined by (B and not C') for the two-particle family occurs, while the physical event defined by B for the one-particle family does not occur. Thus, without the assumption, the relation between quantum events (as mathematical objects) and physical events (to which the frequency interpretation applies) becomes complex.

Next, recall the system first introduced by John Bell. (See the appendix of David Wick's book¹ for an elementary account.) There are two distinguishable spin $1/2$ particles in a certain quantum state. There are quantum events A, B, C associated with the first particle (certain spin components have value $1/2$), and there are quantum events A', B', C' associated with the second particle (the corresponding spin components have value $-1/2$). Each quantum event associated with the first particle is compatible with each quantum event associated with the second particle. Each of A, B, C, A', B', C' have probability $1/2$. Also, each of (A and A'), (B and B'), (C and C') have probability $1/2$. (These probabilities imply that with probability 1, the quantum events A, A' are equivalent, and similarly for the other corresponding pairs.) Finally, each of (A and not B'), (B and not C'), (C and not A') have probability $3/8$.

Imagine many repetitions of the physical situation. First, consider the family generated by B, C' . Consider a repetition in which the physical event defined by (B and not C') relative to this family occurs. In particular, the physical event relative to this family defined by B occurs. Next, consider the family generated by B alone. Then, by the assumption, the physical event defined by B for this family also occurs. In turn, consider the family generated by B, B' . Again, by the assumption, the physical event defined by B occurs. Hence, by the probability prediction, B' occurs. In the same way, consider the family generated by B' alone; again the assumption implies that the physical event defined by B' occurs. Finally, consider the family generated by A, B' . Again, by the assumption, the physical event defined by B' relative to this family occurs. In particular the physical event defined by (A and not B') relative to this family does not occur. The conclusion is that in no repetition is there a simultaneous occurrence of the physical event defined by (A and not B') and of the physical event defined by (B and not C'). In fact, there is never a simultaneous occurrence of two of the three physical events defined by (A and not B'), (B and not C'), (C and not A'). So the frequency of occurrence of at least one of these three physical events must be less than $3/8$. This conclusion contradicts the probability prediction of quantum mechanics.

Some proponents of the consistent-histories theory formulate "rules" of interpretation.

Thus, Griffiths states, “A meaningful description of a (closed) quantum mechanical system, including its time development, must employ a single framework.”² Similarly, Omnès says, “Every description of a physical system should be expressed in terms of properties belonging to a common consistent logic.”³ These rules are extraordinarily obscure. Apparently different descriptions may use different consistent logics, but how are these descriptions related? The rules clearly limit the possibilities of description of a quantum system. Perhaps they could be invoked to claim that a multistage argument, each of whose individual stages is correct, is globally inadmissible. However, such a claim would cast more doubt on the rules than on the argument.

The rules remind us that there is no general notion of conjunction of quantum events. However, the argument presented above uses the conjunction of compatible quantum events, for which there is no problem. The argument does combine physical events, but only according to the following principle. Consider a sequence of repetitions of the physical situation. Suppose that, for each physical event, for each repetition there is a corresponding occurrence or nonoccurrence. Then, for each repetition, for each physical event there is a corresponding occurrence or nonoccurrence. In particular, for each repetition and each pair of physical events, there is or is not a simultaneous occurrence. This mathematical commonplace has nothing to do with quantum mechanics; it is inherent in the frequency interpretation of probability in any domain. Nevertheless, it seems to be at the heart of the issue.⁴

References

1. D. Wick, *The Infamous Boundary: Seven Decades of Heresy in Quantum Physics*, Springer-Verlag (Copernicus), New York (1996).
2. R. B. Griffiths, *Phys. Rev. A* 54, 2759 (1996).
3. R. Omnès, *The Interpretation of Quantum Mechanics*, Princeton U. P., Princeton, N. J. (1994).
4. R. B. Griffiths, *Phys. Rev. A* 57, 1604 (1998); see p. 1615.

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