Additional Problems

Problem A.1: Solve the following initial value problem with Laplace transform.

\[ \frac{d^2 y}{dt^2} - y = \delta(t - 3) \]
\[ y(0) = 0 \]
\[ y'(0) = 0 \]

Problem A.2: Solve the following wave equation.

\[ u_{tt} - a^2 u_{xx} = 0 \quad 0 < x < l, \ t > 0 \]
\[ u(0, t) = u(l, t) = 0 \quad t > 0 \]
\[ u(x, 0) = \varphi(x) \quad 0 \leq x \leq l \]
\[ u_t(x, 0) = \psi(x) \quad 0 \leq x \leq l \]

Problem A.3: Consider the following periodic function with period 6.

To what values does its Fourier series converge to at the following x?
At \( x = -1 \), its Fourier series converges to the value of ____
At \( x = -3 \), its Fourier series converges to the value of ____
At \( x = 3 \), its Fourier series converges to the value of ____
At \( x = 0 \), its Fourier series converges to the value of ____

Problem A.4:

\[ f(x) = \begin{cases} 
    x & \text{if } 0 \leq x \leq 1 \\
    2 - x & \text{if } 1 \leq x \leq 2 
\end{cases} \]

Find the sine series of \( f(x) \).