

Fourier Series, Period  $2L$ 

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left[ a_n \cos\left(n \frac{\pi x}{L}\right) + b_n \sin\left(n \frac{\pi x}{L}\right) \right]$$

$$a_0 = \frac{1}{2L} \int_{-L}^L f(x) dx, \quad a_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(n \frac{\pi x}{L}\right) dx, \quad b_n = \frac{1}{L} \int_{-L}^L f(x) \sin\left(n \frac{\pi x}{L}\right) dx.$$

 Fourier transform of  $f(x)$  :

$$\mathcal{F}[f(x)](k) = \hat{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx.$$

 Laplace transform of  $f(t)$  :

$$\mathcal{L}[f(t)](s) = F(s) = \int_0^{\infty} f(t) e^{-st} ds.$$

$f(x)$	$\mathcal{F}[f(x)](k) = \hat{f}(k)$
$e^{-ax^2}$	$\frac{1}{\sqrt{2a}} e^{-k^2/(4a)}$
$e^{ax} H(x)$	$\frac{1}{\sqrt{2\pi(a+ik)}}$

Formula	Name
$\mathcal{F}[f'(x)] = (ik) \hat{f}(k)$	Differentiation
$\mathcal{F}[f * g] = \sqrt{2\pi} \hat{f}(k) \hat{g}(k)$	Convolution Th.

$f(t)$	$\mathcal{L}[f(t)](s) = F(s)$
1	$\frac{1}{s}$
$t^n$	$\frac{n!}{s^{n+1}}$
$e^{at}$	$\frac{1}{s-a}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\delta(t-a)$	$e^{-as}$
$H(t-a)$	$\frac{e^{-as}}{s}$

Formula	Name
$\mathcal{L}[e^{at} f(t)] = F(s-a)$	s - shifting
$\mathcal{L}[f'(t)] = sF(s) - f(0)$	Differentiation
$\mathcal{L}[f(t-a)H(t-a)] = e^{-as} F(s)$	t - shifting
$\mathcal{L}[tf(t)] = -F'(s)$	Differentiation of $F$