Math 322. Spring 2015
Review Problems for Midterm 1

Chapter 13 (Complex Numbers):
Topic 1: Polar form of complex number .

Question 1.

Let $z = 1 - i$. Evaluate $w = 1/z$ in polar form, with the principal argument.

Question 2.

Let $z_1 = -2 + 2i$ and $z_2 = -6 - 6i$. Evaluate $Arg(z_1/z_2)$.

Topic 2: Operations of complex numbers.
Question 3.

Let $z_1 = 3 + 2i$, $z_2 = 2 - 2i$, find

$$\text{(a)} \quad \frac{z_1 + z_2}{\bar{z}_2^2} \qquad \text{(b)} \quad \text{Im}([(1-i)^8 z_1^2])$$

$$\text{(c)} \quad \left| \frac{z_1 - z_2}{z_2} \right| \qquad \text{(d)} \quad \text{Re}((z_1 + 1)z_2)$$

Topic 3: Roots of complex number .

Question 4.

Find all the solutions for $z^4 = 1$.

Question 5.

Find all the solutions for $z^3 = 2 - 2i$.

Topic 6: Exponential, trigonometric, hyperbolic and logarithmic functions, general power.

Question 6.

Let $z = x + iy$. Find the Re and Im of $e^{1/z}$.

Question 7.

Find the Re, Im and modulus of $e^{-3 + \frac{4\pi}{7}i}$.

Question 8.

Compute $\sin(5 - 2i)$.

Question 9.

Compute $\cosh((n + \frac{1}{2})\pi i)$, where n is an integer.

Question 10.

Show the following identity is true. (Hint: You may need to use the identity $e^{inx} = (e^{ix})^n$).

$$\cos(3\theta) = \cos^3(\theta) - 3\cos(\theta)\sin^2(\theta)$$

Question 11.

Compute $\text{Ln}(5 - 4i)$, $\text{Ln}(-2)$.

Question 12.

Find the principal value of $(1 + i)^{1-i}$.

Chapter 7 (Linear Algebra):
Topic 1: Matrix Operations.

Question 1.

Which of the following equations may not be true? Why not?
(a) $A(BC) = (AB)C$
(b) $(A + B)C = AC + BC$
(c) $(A + B)^2 = A^2 + 2AB + B^2$
(d) $(AB)^T = B^T A^T$

Question 2.

Let

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ 0 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -2 & 3 \end{bmatrix}$$

Calculate the following products or sums or give reasons why they are not defined.
(a) AB (b) BA (c) $A + B$ (d) $A - B^T$

Topic 2: Linear system of equations, row operations

Question 3.

Let

$$A = \begin{pmatrix} -2 & 2 & 6 \\ 1 & -1 & 2 \\ -1 & 1 & 3 \end{pmatrix}.$$

Does the system $Ax = B$ with $B = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ admit solutions? If so, how many? Find them.

Question 4.

Let

$$A = \begin{bmatrix} 0 & -6 & 4 \\ 1 & -2 & -2 \\ 1 & -8 & 2 \\ 3 & -12 & -2 \end{bmatrix}$$

Let $b = [1, 2, 3, 7]^T$. Does the following system of equations have solution(s)? If your answer is yes, find the general form of the solution(s).

$$Ax = b.$$