

### Math 322 Section 3 Written Homework 3

1) The delta function  $\delta(x)$  is defined to be the a function such that for any function  $f(x)$ ,

$$\int_a^b \delta(x) f(x) dx = \begin{cases} f(0) & \text{if } a < 0 < b \\ 0 & \text{if } a \geq 0 \text{ or } b \leq 0 \end{cases}$$

Actually, it can be shown that no such function exists, but it still has a Fourier series since, for instance,

$$\int_{-\pi}^{\pi} \delta(x) \cos 3x dx = \cos(0) = 1.$$

1) Calculate  $\int_a^b \delta(x - \frac{\pi}{2}) f(x) dx$  for any function  $f(x)$ .

2) Write the Fourier series for the delta function  $\delta(x - \frac{\pi}{2})$  gotten as an odd periodic half range expansion on the interval  $[0, \pi]$ .

3) Show that the series does not converge for  $x = \frac{\pi}{2}$ .

4) Use separation of variables to find the solution to the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

where  $u(x, t)$  satisfies  $u(0, t) = u(\pi, t) = 0$  and  $u(x, 0) = \delta(x - \frac{\pi}{2})$ .

5) Explain why we say that the heat equation shrinks high frequency modes faster than low frequency modes.