Written Homework 4

1) Find the complex Fourier series of the following functions

$$g(x) = \sin x \qquad -\pi < x < \pi$$
$$h(x) = \begin{cases} 0 & \text{if } -1 < x \leq 0 \\ 1 & \text{if } 0 < x < 1 \end{cases}$$

2) If the complex Fourier series of the function $f(x) = e^x$, where $-\pi < x < \pi$, is given by

$$F(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$$

then give the coefficients c_0, c_1, c_{-1}, c_3.

3) Convert the differential equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$
$$u(x, 0) = f(x)$$

the function $u(x, t)$ to a differential equation for $\hat{u}(w, t)$.

4) Write an integral form of the solution to the heat equation on the whole real line with initial condition

$$u(x, 0) = \begin{cases} 1 & \text{if } |x| \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

5) Find the solution to the heat equation on the whole real line with initial condition $u(x, 0) = \delta(x - 1) + \delta(x + 1)$.