

# Chapter Check for Chapter 1

September 16, 2015

1. Let  $\mathcal{F}(X, F)$  denote the set of functions from the set  $X$  to the field  $F$ .

a. Describe the usual vector space structure for  $\mathcal{F}(X, F)$  (you do not have to show it is a vector space).

b. Let  $x_0 \in X$  and for each  $a \in F$ , consider the set  $S_a = \{f \in \mathcal{F}(X, F) : f(x_0) = a\}$ . For which values of  $a$  is  $S_a$  a subspace? Justify your answer.

c. If  $X$  is a finite set, show that the dimension of  $\mathcal{F}(X, F)$  is equal to the number of elements in  $X$ .

2.

a. Find a basis for  $\mathbb{R}^3$  that contains the vector  $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ . Justify your answer.

b. Is the set  $\left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix} \right\}$  a basis for  $\mathbb{R}^3$ ? Why or why not?

3. (Comprehensive/graduate option only) Consider the subspace  $W = \{(t, t, t) \in \mathbb{R}^3 : t \in \mathbb{R}\}$  of  $\mathbb{R}^3$ . Give an explicit basis for  $\mathbb{R}^3/W$ . Justify your answer.