Homework problem from Chapter 5

Let A be a square matrix with real entries. We can define a matrix $\exp A$ by the "power series"

$$\exp A = I + A + \frac{1}{2}A^2 + \frac{1}{3!}A^3 + \cdots$$

Suppose A is diagonalizable with eigenvalues $\lambda_1, \ldots, \lambda_n$ (not necessarily distinct).

a. Give a general formula for A^n of the form $Q^{-1}D_nQ$ where D_n is diagonal. Be sure to specify what Q and D_n are precisely.

b. Give a general formula for $\exp A$ of the form $Q^{-1}DQ$ where D is diagonal.

c. Calculate $\exp \begin{pmatrix} 3 & 12 \\ 0 & -1 \end{pmatrix}$.