

Problem Set 1

Math 466

due February 5, 2008

1. Your instructor hands out 4 essay questions and states that 2 will be on the midterm exam. If you have time to prepare for only 2 of them. For $x = 0, 1$ and 2 , what is the probability that you will have prepared for x questions that actually appear on the exam.
2. A box contains 30 red balls, 30 white balls, and 30 blue balls. 10 balls are chosen at random, without replacement. What is the probability that at least one color is missing?
3. Find the probability that *exactly one* of the three events A_1, A_2, A_3 occur.
4. Compute the median of an exponential random variable.
5. Find the density of e^X where X is a normal distribution.
6. Let X_1, \dots, X_n be independent standard normal random variables, then $X_1^2 + \dots + X_n^2$ is a continuous random variable with a χ_n^2 distribution.
7. Let X and Y have the following mass function.

$Y \setminus X$	0	1	2	3	4
0	0.08	0.07	0.06	0.01	0.01
1	0.06	0.10	0.12	0.05	0.02
2	0.05	0.06	0.09	0.04	0.03
3	0.02	0.03	0.03	0.03	0.04

Find

- (a) $P\{X > Y\}$.
 - (b) The marginal mass functions for X and Y .
 - (c) The conditional mass function $p_{Y|X}(y|2)$
 - (d) $E[Y^2|X = 2]$.
8. Let X and Y have joint density function

$$f(x, y) = \begin{cases} c(x^2 + y) & \text{for } 0 \leq y \leq 1 - x^2, \\ 0 & \text{otherwise} \end{cases}$$

Determine

- (a) the value of the constant c ,
 - (b) the marginal density for X ,
 - (c) $P\{0 \leq X \leq 1/2\}$.
 - (d) the conditional density $f_{Y|X}(y|x)$, and
 - (e) $E[Y|X = x]$.
9. Compute the mean, the variance, and the generating function of a Poisson random variable.
10. Compute the mean and the variance of a $\Gamma(1, \beta)$ random variable. Pick two parameters values for β , simulate 1000 Γ random variables and report their sample mean.
11. Compute the moment generating function of a standard normal random variable. Use this to find its first 4 moments.