Math 464 Fall 2012 Homework #6

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Due 10/18

Recall we defined the following in class on Tuesday 10/16: let X and Y be two random variables with joint density f(x,y), and let x be any real number. The conditional density of Y given X=x is defined to be

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} \tag{1}$$

where $f_X(x)$ is the marginal density

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y) \ \ dy \ \ . \tag{2}$$

Exercise. Suppose

$$f(x,y) = \begin{cases} \frac{12}{5}x(2-x-y), & 0<x<1 \text{ and } 0<y<1 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

Find (a) the marginal density $f_Y(y)$; (b) the conditional density of X given that $Y = 1/2$; and (c) the conditional density of X given that $Y = y$ for all $y \in (0,1)$.

Answers:

(a) $f_Y(y) = \frac{12}{5}\left(\frac{2}{3} - \frac{y}{2}\right)$

(b) $f_{X|Y}(x|1/2) = \frac{6x(3/2-x)}{5/2}$

(c) $f_{X|Y}(x|y) = \frac{6x(2-x-y)}{4-3y}$

Problem. Suppose the joint density of X and Y is

$$f(x,y) = \begin{cases} e^{-x/y}e^{-y}/y, & 0<x<\infty \text{ and } 0<y<\infty \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Find (a) the conditional density of X given that Y=y, and (b) $P(X>1|Y=y)$ for all $y>0$.