Math 322 Sect. 2
Homework #10

Due Wed. 4/1/15

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Please write these up and turn them in.

1. Consider the system $X'(t) = AX(t)$ with

$$\begin{bmatrix} 5 & 0 & 6 \\ 3 & 3 & 7 \\ -3 & 0 & -4 \end{bmatrix} \tag{1}$$

   By diagonalizing A, find the solution with initial conditions

$$X(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} . \tag{2}$$

2. Consider the scalar differential equation

$$x'''(t) + ax''(t) + bx'(t) + cx(t) = 0. \tag{3}$$

   (a) By introducing two new variables, rewrite the equation as a first-order linear system of the form

$$X'(t) = AX(t) \ , \quad X(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} , \tag{4}$$

   y and z are the new variables you defined.

   (b) Suppose now $a = b = c = 0$. Find the general form of solutions to Eq. (3).

   (c) When $a = b = c = 0$, is the matrix A you found in (a) diagonalizable?

   (d) For $a = b = c = 0$, compute the matrix exponential e^{tA} directly from the power series definition. Using this, find a general solution for Eq. (4). Is it equivalent to your solution in (b)?