lec9

September 12, 2016

Exam 1 review.
Recall that if a continuous function f is *increasing* on an interval $[a,b]$, then we have

$$LEFT(n) \leq \int_a^b f(x)\ dx \leq RIGHT(n)$$

and if the function is *decreasing* then

$$RIGHT(n) \leq \int_a^b f(x)\ dx \leq LEFT(n).$$

If the function is *concave up,* then

$$MID(n) \leq \int_a^b f(x)\ dx \leq TRAP(n)$$

and if *concave down,* then

$$TRAP(n) \leq \int_a^b f(x)\ dx \leq MID(n).$$

These statements are always true regardless of whether the function is positive or negative. The examples below illustrate this.

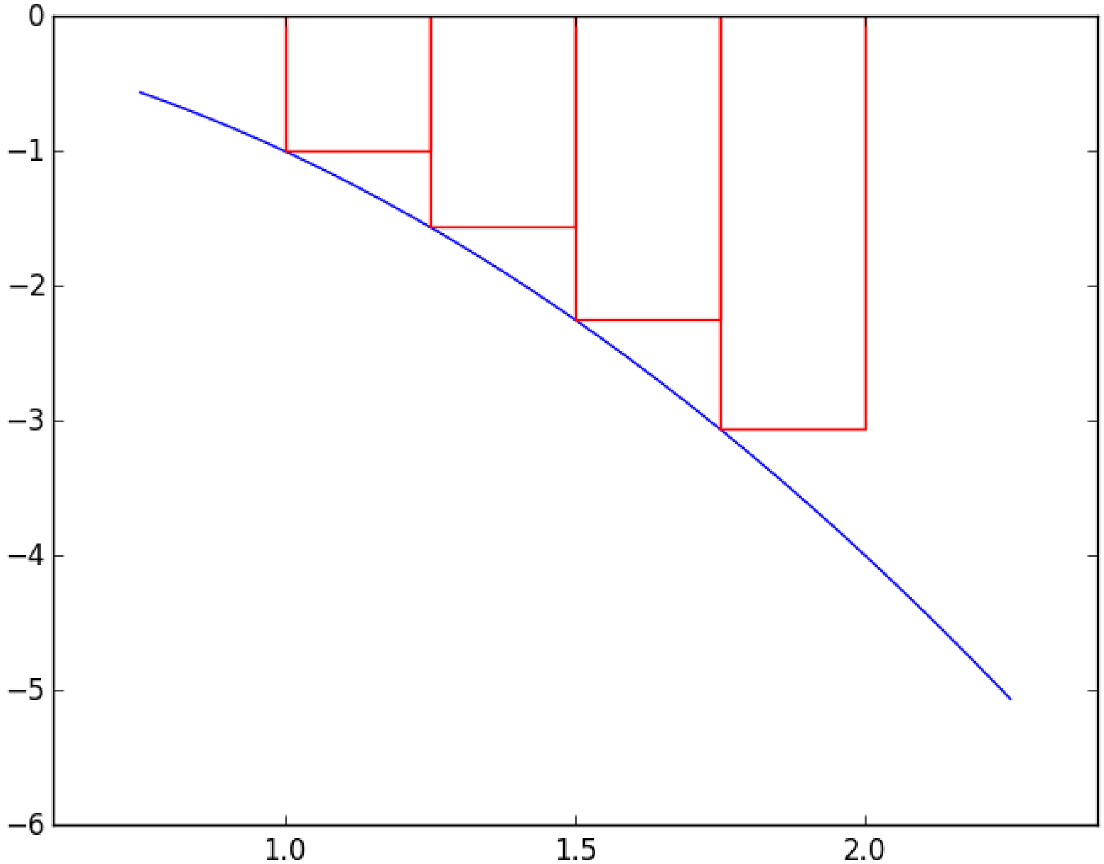
```
In [2]: ## You can ignore this.
        using lec8,PyPlot
```

Ex. Use LEFT(n) to approximate $-\int_1^2 x^2\ dx = -\frac{7}{3}$.

```
In [5]: ## The true answer
        -7/3

Out[5]: -2.3333333333333335

In [4]: left(x->-x^2,a=1,b=2,n=4)
```




```
LEFT(4) = f(1.0)*0.25 + f(1.25)*0.25 + f(1.5)*0.25 + f(1.75)*0.25
```

```
Out[4]: -1.96875
```

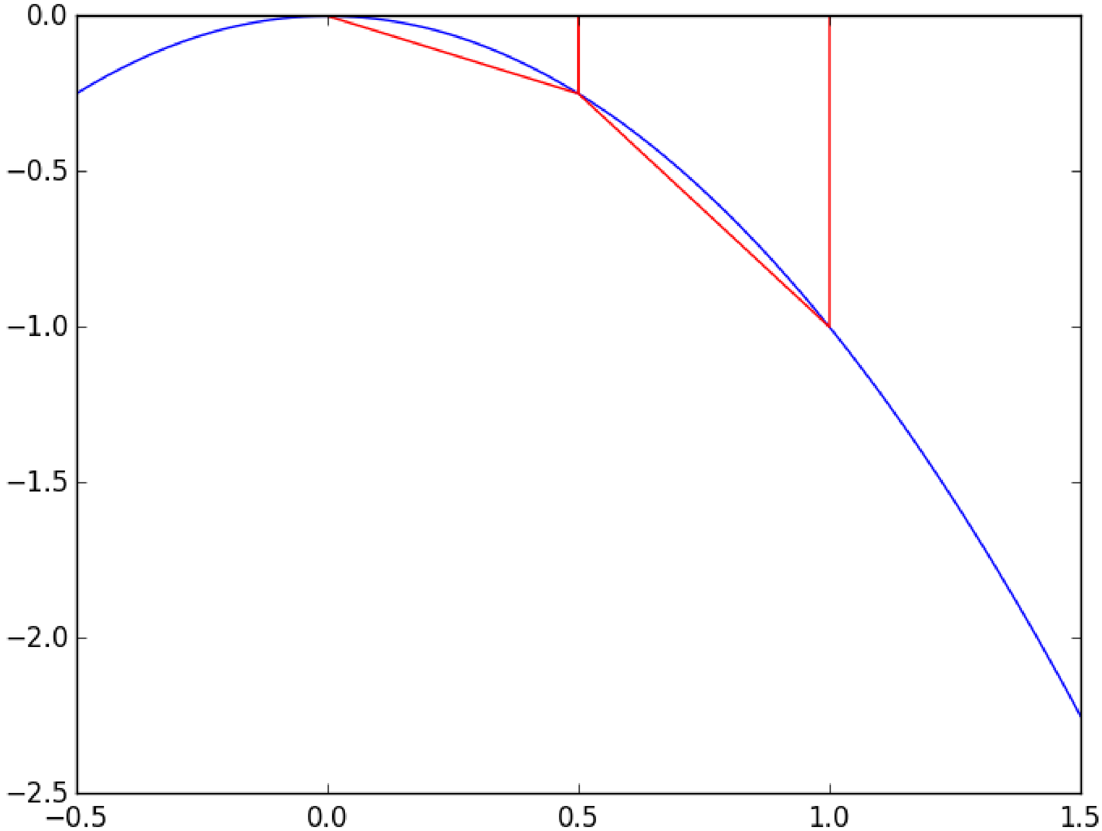
Observe that the LEFT(4) approximation gives -1.97, whereas the true answer is about -2.33. Since the approximation is *less negative* than the true answer, we would still say that LEFT(4) is an **overestimate**.

You might have been tempted to call LEFT(4) an underestimate in this case because the total *area* of the rectangles (about 1.97, i.e., the absolute value of the LEFT(4) approximation) is indeed smaller than the area enclosed by the curve, the x axis, and the vertical lines $x = 1$ and $x = 2$. But for the actual integrals (which are negatives of the areas in this case) LEFT(4) is an overestimate.

Similarly, if we apply TRAP(n) or MID(n) to a function that can be negative, the above rules about overestimating and underestimating still apply.

Ex. Use TRAP(n) to approximate $-\int_0^1 x^2\ dx = -\frac{1}{3}$.

```
In [6]: trap(x->-x^2,a=0,b=1,n=2)
```




```
TRAP(2) = 0.5*[f(0.0)+f(0.5)]*0.5 + 0.5*[f(0.5)+f(1.0)]*0.5
```

Out[6]: -0.375

```
In [8]: ## True value.
        -1/3
```

Out[8]: -0.3333333333333333

Again, because the function is concave down, we know TRAP(2) should give an underestimate. It may not look like it from the picture, but because the function is negative, TRAP(2) is indeed more negative than the true value.

In [ ]: