

# Math 254-009 A short glossary for 1st and 2nd-order equations

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Last revised: September 12, 2016

- **Autonomous:** A first-order ODE is autonomous if it has the form

$$\frac{dy}{dx} = f(y), \quad (1)$$

i.e, the right hand side does not contain the independent variable.

- **Homogeneous:** A linear second-order equation is homogeneous if it has the form

$$a_2(x)y''(x) + a_1(x)y'(x) + a_0(x)y(x) = 0. \quad (2)$$

It is **non-homogeneous** or **inhomogeneous** if it has the general form

$$a_2(x)y''(x) + a_1(x)y'(x) + a_0(x)y(x) = F(x). \quad (3)$$

(In the above, the coefficients  $a_2(x)$ , etc., are given functions of  $x$ .)

- **Linear:** A first-order equation is linear if it can be written in the form

$$a_1(x)y'(x) + a_0(x)y(x) = F(x). \quad (4)$$

(The coefficients  $a_1(x)$  and  $a_0(x)$ , as well as the right hand side  $F(x)$ , are all given functions of  $x$ .) A second-order equation is linear if it can be written in the form

$$a_2(x)y''(x) + a_1(x)y'(x) + a_0(x)y(x) = F(x). \quad (5)$$

- **Ordinary differential equation (ODE):** A general 1st-order ODE has the form

$$y'(x) = f(x, y(x)). \quad (6)$$

A general 2nd-order ODE has the form

$$y''(x) = f(x, y(x), y'(x)). \quad (7)$$

- **Initial value problem (IVP):** An IVP is an ODE together with a number of initial conditions (also known as initial values). A 1st-order IVP has the form

$$y'(x) = f(x, y(x)) , \quad y(x_0) = Y_0. \quad (8)$$

A 2nd-order IVP has the form

$$y''(x) = f(x, y(x), y'(x)) , \quad y(x_0) = Y_0 , \quad y'(x_0) = Y_1. \quad (9)$$