Lecture 9 notes

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Today we discussed

- The period of a chain.
- The problem of convergence to equilibrium.
- Introduced the concept of long-time averages.

I largely followed Sect. 1.6 of the text. (We will return to Sect. 1.5, on the detailed balance condition, later.)

Slight deviations from the text:

1) For the Ehrenfest chain, it can be made clear that the period is 2 by grouping the states into two groups, the even and the odd. Then one sees that even though transitions are random, the chain *deterministically* alternates between even and odd states.

2) The Ehrenfest example, seen in this way, generalizes: any chain in which the states form d groups, and where the chain cycles through the groups in a deterministic, cyclic fashion must have period d.

3) In the proof of Lemma 1.17, instead of proof by contradiction, we made use of the following fact: if I is a set of positive integers and $n|m$ (i.e., n divides m) for all $m \in I$, then n divides the greatest common divisor $\gcd(I)$.