Notes for Lecture 27 and 28

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These two lectures are a brief introduction to continuous time Markov chains (CTMCs). Every I covered is in Sections 4.1 and 4.2 of the text. Without going into details of proofs etc., I covered

- Markov property for CTMCs;
- transition probabilities;
- Chapman-Kolmogorov equations;
- transition rates;
- Kolmogorov's forward and backward equations; and
- stationary distrbutions.

All these are explained in the text. I'll just mention one deviation: I defined

$$q(i,j) = \frac{d}{dt} p_t(i,j) \tag{1}$$

even when $i = j$. This is not what the book does, and is not quite standard, but it's convenient.

For completeness I list my main examples:

1) Poisson process $N(t)$.

2) A machine has two states, UP or DOWN. It breaks down at a rate of once a day. When this happens, a repair person is called, and with an exponential time of rate 9, they will fix the machine. On average, what fraction of time is the machine up and running?

3) My parents pay unannounced viists once a month, and each time stay an exponential amount of time with mean 1/2 month. My sister does the same, but visiting 3 times a month and stays on average 1/4 a month. The two processes are independent. What fraction of the time are they both visiting me?