Notes for Lecture 29

klin@math.arizona.edu

May 5, 2020

Final exam. Today we discussed the final exam, which will be comprehensive. You will be responsible all the topics that were on the midterm (basically most of Ch. 1), as well as the sections of Ch. 2 we covered (Homework 9, 10, and 11) and the material regarding continuous time Markov chains on Homework 12. I will send out more detailed information separately.

Random topics (pun intended). I asked if any of you had questions. We briefly touched on a few topics. I want to give you references in case any of you are interested.

- *Martingales.* Ch. 5 in our textbook discusses basic properties of martingales, and will hopefully give you a feel for why it's useful. This material is used in Ch. 6 on mathematical finance. Those of you interested are encouraged to flip through those pages. I'm happy to answer any *math*[1] questions you have!
- *Brownian motion.* One of you asked about Brownian motion and applications to finance. Again, I don't know much about finance. I do know a little bit about the physical phenomenon of Brownian motion and its mathematical model, called the "Wiener process." These models are examples of *Gaussian processes,* stochastic processes X_t for which the joint distributions $X_{t_1}, \cdots, X_{t_n}$ are Gaussian for all n and $t_1 < \cdots < t_n$.

  Those of you who are interested can read up on the basics of Gaussian processes in Ch. 4–6 of the supplemental text by Hoel, Port, and Stone. Those of you with a little basic physics background may enjoy *Investigations on the Theory of the Brownian Movement* by Einstein. (These are English translations of his 1905 papers on Brownian motion.)

[1] I know next to nothing about finance.

- *Metropolis algorithm.* This is an example of a "Markov chain Monte Carlo" algorithm. One of its main applications for estimating expectation values. For example, suppose one has a (very large finite or countably infinite) set S, a probability distribution π on S, and a function φ, and wants to estimate the expected value

  $$\sum_{x\in S}\varphi(x)\pi(x). \tag{1}$$

  If S is large and/or if the function φ is computationally expensive to evaluate (for example, imagine S being a large collection of images, each x being one image, and $\varphi(x)$ being a metric measuring the performance of a neural network deciding whether image x contains a cat), then it may be impractical to evaluate the sum exactly. One strategy would be to *design* a Markov chain X_n that is easy to simulate and has π as its stationary distribution. Then

  $$\frac{1}{N}\sum_{n=1}^{N}\varphi(X_n) \tag{2}$$

  would provide a reasonable approximation of Eq. (1) for large N.

  If you're interested in this topic, there's a short section (Sect. 1.5.2) explaining more of the mathematical foundation. For a practical guide, search for "Introduction to Monte Carlo methods" by David MacKay.