An Alternative Language Learning Model

Project Description

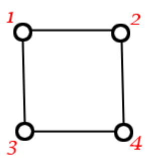
- Often children learn to speak the language of their peers as opposed that of their parents.
- It is thought that a model where the learners randomly move about a graph consisting of nodes each of which represents a distinct language, may better model this learning than the traditional teacher-student model. [1]

Scientific Challenges

- Looking at this alternative model strictly from a mathematical standpoint it is interesting to explore which *K* values lead to convergence.
- Understanding the dynamics of a model in which learners move from node to node on a graph. In particular, understand why convergence can be reached with a very low *K* value.

Potential Applications

- As mentioned in [2] children in a Nicaraguan school for the deaf developed their own sign language completely aside from what was being taught to them at school by their teachers. It may be possible to use this model to understand such an occurrence.



0	1	1	0
1	0	0	1
1	0	0	1
0	1	1	0

Figure 1: a graph and its matrix representation

row #	adjacent nodes	
1	2	3
2	1	4
3	1	4
4	2	3

Figure 2: the matrix which stores the list of adjacent nodes for each node (corresponding to figure 1 in this case). Highlighted area is actual matrix.

Team Members:

Ryan James Hatch
Adam Michael Rinne

Methodology

- The languages are modeled as nodes on a graph.
- A matrix of ones and zeros is constructed to represent the graph with N nodes, each with its number of adjacent nodes between n and 2*n.
- Each row of the matrix represents the corresponding node. An entry of one in position (i,j) indicates that nodes i and j are connected. An entry of zero indicates the two nodes are not connected.

Numerical Setup

- From this matrix a list of adjacent nodes is created for each node. These lists are stored as rows in a new matrix where the row number corresponds to the node number and the entries in the row are the adjacent node numbers.
- A vector is created where the indices of the vector indicate the node numbers and the entries indicate the number of learners at that node.
- Initially each of the L learners is placed at an empty node, that is to say that no two learners occupy the same node to start with.

Evolution of the Numerical Simulation

- In one timestep, or iteration, each learner who needs to be moved is moved exactly once.
- Each learner can only move to a node adjacent to her current node. This adjacent node is randomly chosen from the list of adjacent nodes *(figure 2)* for her current node.
- Each learner continues to move until she arrives at a node at the same timestep as at least *K – 1* other learners, or until she arrives at a node with an existing cluster of *K* or more learners.
- The simulation continues until the *K* value is satisfied at each node; that is, each node has either 0 or at least *K* learners.

Output of the Numerical Simulation

- When the point is reached where the *K* value is satisfied at each node, the vector containing the number of learners at each node is checked for convergence in the following manner: The number of non-zero entries in the vector is determined. If this number is equal to one, there is convergence. If the number is greater than one, there are two or more nodes that each have *K* or more learners; thus no convergence.

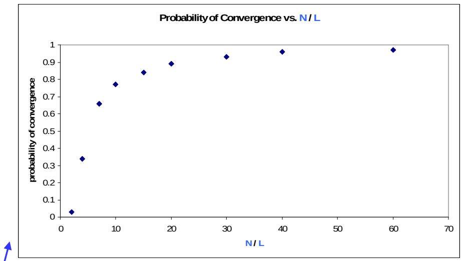
Results

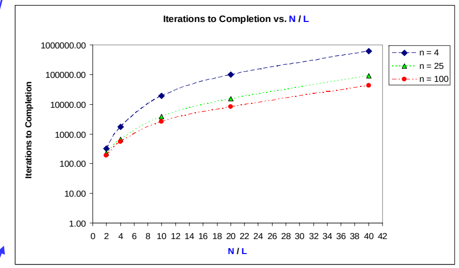
Keeping L = 50 and n = 25 as N/L ⟶ ∞ the probability of convergence increases towards the limit of 1.0

Again, keeping L = 50 we compare the number of iterations to N / L. This was done for three significantly different values of n and plotted on the same graph for comparison.

Glossary of Technical Terms

K – the aspiration level, i.e. a learner continues to move until she reaches a node at the same time as *K-1* other learners.
edge – a path that can be used in both directions between exactly two nodes.
adjacent – two nodes are said to be adjacent if they are connected via an edge.
convergence – the case when satisfaction of the K-value at each node leads to all learners occupying the same node.
N – Total number of nodes on the graph
n – for each node, n ≤ # adjacent nodes ≤ 2*n
L – the total number of learners on the graph





References

1. Matsen, Frederick & Nowak, Martin. *Win-stay, Lose-shift in Language Learning from Peers*. PNAS **101**, 18053 – 18057 (2004)
2. "Nicaraguan deaf children create a new sign language entirely their own" Child Health News 18-Sep-2004 <http://www.news-medical.net/?id=4883>

Acknowledgments

This project was mentored by Silvia Madrid Jaramillo, whose help is acknowledged with great appreciation.
Support from a University of Arizona TRIF (Technology Research Initiative Fund) grant to J. Lega is also gratefully acknowledged.

Analyzing the Red Grouse – *T. tenuis* System

Denise Brown Ryan Humphrey Jessica Ryder David Sonenschein

Project Description

- England's red grouse populations undergo periodic population cycles.
- Hudson et al. hypothesize that the fluctuations are due to the presence of a parasite [2].
- Modeling the grouse-parasite system allows us to understand the cyclic behavior and determine a possible solution to stabilize the fluctuations.
- Our goals are to understand the authors' model and to reproduce their numerical results.

Scientific Challenges

- Learning to develop an analytical model to describe a complex real-life problem

Potential Applications

- Predict the number of hosts which must be treated to prevent population crashes
- Model other predator-prey or host-parasite systems

Methodology

- Study mathematical concepts of predator prey systems: stability and phase plane analysis
- Examine the authors' preliminary models to better understand the parasite's effect on grouse mortality and fecundity
- Understand and numerically analyze a 6-dimensional host-parasite model

Glossary of Technical Terms

Trophic interactions: Interactions in a biological system involving eating or being eaten [2].
Hypobiosis: A period of arrested development between the larval and adult stages of a parasite's life [1].

Acknowledgments

We would like to thank our mentor Julia Arciero for her help on this project. We are also grateful for support from a University of Arizona Technology Research Initiative Fund grant to J. Lega.

Model

1. $$\frac{dH_U}{dt} = (1-p)\theta C - [b + \Delta(H_U + H_T)]H_U - \alpha P_U$$

(i) (ii) (iii)

(i) The number of untreated chicks that become untreated adult hosts.
(ii) A combination of the death of the untreated hosts and the decrease in fecundity and survival rates of the untreated host due to population density.
(iii) The deaths of untreated hosts caused by parasites.

2. $$\frac{dC}{dt} = a(H_U + H_T) - \delta(P_U + P_T) - (b + \theta)C$$

(i) (ii) (iii)

(i) The birth of chicks given grouse fecundity .
(ii) The reduction in grouse fecundity due to the parasite.
(iii) The death rate of the grouse and the proportion of chicks that reach adulthood.

3. $$\frac{dP_T}{dt} = \beta H_T W - \{[if\,(dP/dt) > 0, \mu + c, \mu] + b + \alpha\}P_T - \alpha \frac{p_T^2}{H_T}\frac{k+1}{k}$$

(i) (ii) (iii)

(i) The growth of free-living parasites into adults.
(ii) This term is similar to the second term in (4) below, but takes into account parasite deaths in treated hosts due to the anthelmintic, where $P = P_T + P_U$.
(iii) The parasite interaction inside the host (competition between parasites).

4. $$\frac{dP_U}{dt} = \beta H_U W - (\mu + b + \alpha)P_U - \alpha \frac{p_U^2}{H_U}\frac{k+1}{k}$$

(i) (ii) (iii)

(i) The growth of free-living parasites into adults.
(ii) This term is similar to the second term in (IV) below, but takes into account parasite deaths in treated hosts.
(iii) The parasite interaction inside the host (competition between parasites).

5. $$\frac{dW}{dt} = \lambda(P_U + P_T) - [\gamma + \beta(H_U + H_T)]W$$

(i) (ii)

(i) The birth rate of the parasite.
(ii) A combination of the death rate and transmission rate of the free-living parasites.

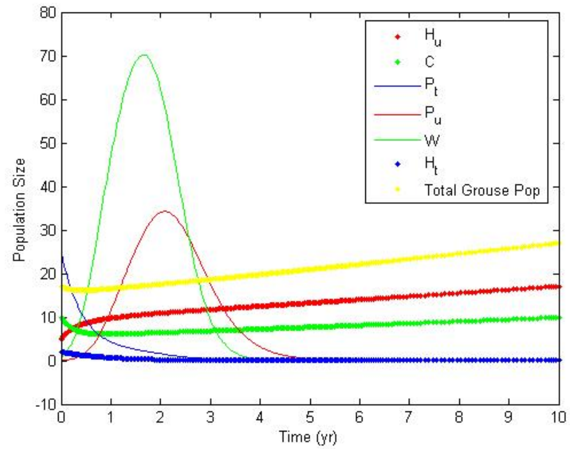
6. $$\frac{dH_T}{dt} = p\theta C - [b + \Delta(H_U + H_T)]H_T - \alpha P_T$$

(i) (ii) (iii)

(i) The number of treated chicks that become treated adult hosts.
(ii) A combination of the death of the treated hosts and the decrease in fecundity and survival rates of the treated host due to population density.
(iii) Deaths of treated hosts caused by parasites.

Results

Fig. 1. Our solution to the six-equation model, plotted in MATLAB, p = 0.



- Our results do not agree with Hudson et. al's when using the same parameters
- The total grouse population increases over time because the numbers of untreated hosts and chicks are both increasing
- There is an initial increase in the numbers of free-living parasites and parasites in untreated hosts, but both die off because the number of grouse is still not sufficient to support the parasite population
- Hudson et al.'s work shows that treating approximately 20% of total grouse is sufficient to prevent population crashes

References

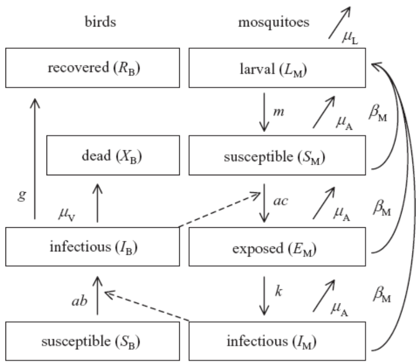
[1]. A. P. Dobson, P. J. Hudson, *Regulation and Stability of a Free-Living Host-Parasite System: Trichostrongylus tenuis in Red Grouse. II. Population Models*, The Journal of Animal Ecology **61 No. 2**, 487-498 (1992).
[2]. P. J. Hudson, A. P. Dobson, D. Newborn, *Prevention of Population Cycles by Parasite Removal*, Science **282**, 2256-2258 (1998).
[3]. *Two-species models*, Math 485/585 Class Notes (compiled by J. Lega), University of Arizona Dept. of Mathematics, 105-113 (Spring 2005).
[4]. P. J. Hudson, D. Newborn, A. P. Dobson, *Regulation and Stability of a Free-Living Host-Parasite System: Trichostrongylus tenuis in Red Grouse. I. Monitoring and Parasite Reduction Experiments*, The Journal of Animal Ecology **61 No. 2**, 477-486 (1992).
[5]. A. A. Berryman, *Lotka-Volterra Model*, Washington State University, <http://classes.entom.wsu.edu/543/LVmodel.htm> accessed 25 April 2005.
[6]. P. J. Hudson, A. P. Dobson, D. Newborn, Cyclic and Non-Cyclic Populations of Red Grouse: A Role for Parasitism?, Ecology and Genetics of Host-Parasite Interactions, 55 No. 21, 77-89 (1985).

Mathematical Epidemiology:
Modeling the Spread of the West Nile Virus

Project Description
- West Nile virus is a disease spreading globally and emerging locally.
- An extended SIR model is needed to describe the spread of the virus among mosquito and bird populations.[1][2]
- The goal of the project is to see how modeling techniques are implemented in current epidemiological research.

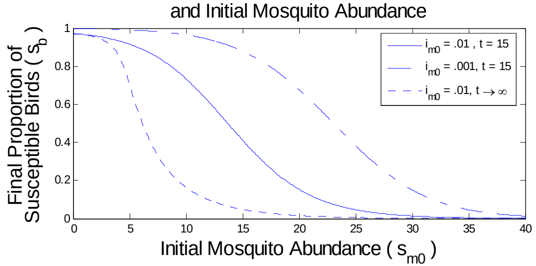
Scientific Challenges
- The project involves a dual species dependence, between vectors (mosquitoes) and hosts (birds), therefore the dimension of the system is large and thus more difficult to conceptualize.
- The model utilizes a new approach, based on current analytical research, to help determine the necessary indicators to control mosquito populations

Potential Applications
- Modeling the virus allows prediction of how the virus will affect different populations in different geographical regions.
- The model will also help in determining how best to prevent serious public health consequences.



A compartmental extended SIR model for West Nile cross-infection between birds and mosquitoes reproduced from [3]. This compartmental diagram is used to develop the model's system of differential equations.

Relationship Between Final Susceptible Crow Population and Initial Mosquito Abundance



Team Members:
Joe Aldridge, Optical Sciences and Engineering
Katie Moore, Astronomy
Shaheed A. Shabazz, Mathematics and Mechanical Engineering

Methodology
1. The team selected an appropriate model (system of differential equations) which accurately describes the cross-infection of the West Nile virus between birds and mosquitoes. The model consists of eight compartments defined by the following:
   a) *Susceptible* — describes the segment of the bird and mosquito populations that are currently susceptible to the virus
   b) *Infectious* — describes the segment of the bird and mosquito populations that are currently infected with the virus and able to infect others
   c) *Recovered* — describes the segment of the bird population that has recovered from the virus
   d) *Dead* — describes the segment of the bird population that died from the virus
   e) *Larval* — describes the mosquito population in its larval stage of development
   f) *Exposed* — describes the segment of mosquito population that has been exposed to the virus, but not infected with the virus

2. To avoid complexity, the team wrote the system of equations in dimensionless form, to eliminate non-essential parameters.

3. The team determined the DFE solution set to be as follows:
   a) The bird equilibrium values are determined to be $(s_{b0}, i_{b0}, r_{b0}, x_{b0}) = (1,0,0,0)$. This reflects that all of the birds are susceptible, and none have been in contact with the disease.
   b) The mosquito equilibrium values are determined to be $(l_{m0}, s_{m0}, e_{m0}, i_{m0}) = (\varphi_{m0} s_{m0}/\eta+\mu_l, s_{m0}, 0, 0)$. The value l_{m0} is determined by setting the larval and susceptible mosquito differential equations to zero, substituting known parameter and equilibrium values, and solving. The value s_{m0} is determined by first determining R_0, setting it equal to unity, and solving.

4. To observe the behavior, the team used parameter values to simulate the model's progression with time.

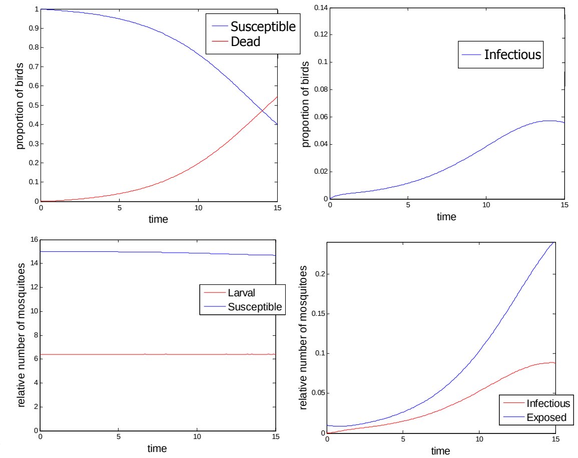
Results
1. We determined an expression for $\mathbf{R_0}$, which indicates whether a disease will become an epidemic.

2. The initial proportion of susceptible mosquitoes indicates the resulting level of invasiveness.

3. Killing mosquitoes slows the spread of West Nile; killing birds speeds it up.

Glossary of Technical Terms
SIR: The Susceptible-Infectious-Recovered model for studying populations with disease.
DFE: The Disease Free Equilibrium. The fixed point for the system at which the population is free of disease [2].
R_0: The basic reproduction number. The average number of secondary infections resulting from having a primary infection introduced into a completely susceptible population.

Case of Moderate Outbreak
$[s_b\ i_b\ r_b\ x_b\ l_m\ s_m\ e_m\ i_m] = [1\ 0\ 0\ 0\ l_{m0}\ s_{m0}\ 0\ 0.01]$, where $s_{m0} = 7.5$ and l_{m0} depends on s_{m0} as $l_{m0} = \phi_m s_{m0}/(\eta + \mu_l)$.



References
[1] Brauer, F. & Castillo-Chavez, C. 2001 *Mathematical Models in Population Biology and Epidemiology.* Springer.
[2] van den Driessche, P. & Watmough, J. 2002 Reproduction Numbers and Sub-Threshold Endemic Equilibria for Compartmental Models of Disease Transmission. *Math. Biosci.* **180,** 29-48.
[3] Wonham, M.J., de-Camino-Beck, T. & Lewis, M.A. 2004 An Epidemiological Model for West Nile Virus: Invasion Analysis and Control Applications. *Proc. Royal Society of London* **271,** 501-507.

Acknowledgments
This project was mentored by Suzanne Robertson, whose help is acknowledged with great appreciation. Support from a University of Arizona TRIF (Technology Research Initiative Fund) grant to J. Lega is also gratefully acknowledged.

Parallel Parking and RSA

Project Description

- Our model investigates how the gap size distribution of cars parked in parallel relates to the process of RSA.
- Our model utilizes the everyday concept of car parking to illustrate how the process of RSA occurs at the microscopic or macroscopic level.
- Model A of [1] simulates RSA with the modification of requiring a fixed amount of space for maneuvering of vehicles into parking spots.
- Model B of [1] simulates car parking using RSA with the additional condition that after adsorption, a car will pull forward to the next nearest car with a given probability *p*.
- Goals:
  - ❑ Reproduce models from [1].
  - ❑ Collect empirical data and compare to results of models in [1].
  - ❑ Use computer simulation of car parking and compare to results of models in [1] and collected empirical data.

Scientific Challenges

- Use car parking model to understand the application of RSA as a way to model protein adsorption on a cell membrane.

Potential Applications

- Provide insight for city planning and development.

$$\frac{\partial p(t)}{\partial t} = k_a \Phi(t)\,, \qquad (1)$$

$$\Phi(t) = \int_\sigma^\infty (x-\sigma)c(x,t)dx \qquad (2)$$

$$p(t) = \int_0^\infty c(x,t)dx\,. \qquad (3)$$

$$1-p(t)\sigma = \int_0^\infty xc(x,t)dx\,. \qquad (4)$$

$$\frac{\partial c(x,t)}{\partial(k_a t)} = -H(x-\sigma)(x-\sigma)c(x,t) + 2\int_{x+\sigma}^\infty c(x',t)dx'\,, \qquad (5)$$

$$c(x,t) = \frac{F(k_a t\sigma)}{\sigma^2}\exp(-k_a(x-\sigma)t) \qquad (6)$$

$$F(t) = t^2\exp\left(-2\int_0^t \frac{1-e^{-u}}{u}du\right). \qquad (7)$$

where:

σ is the car length
$\Phi(t)$ is the insertion probability at time t
$p(t)$ is the number of cars parked per length
$H(x)$ is the unit step function
$c(x,t)$ is the gap size distribution

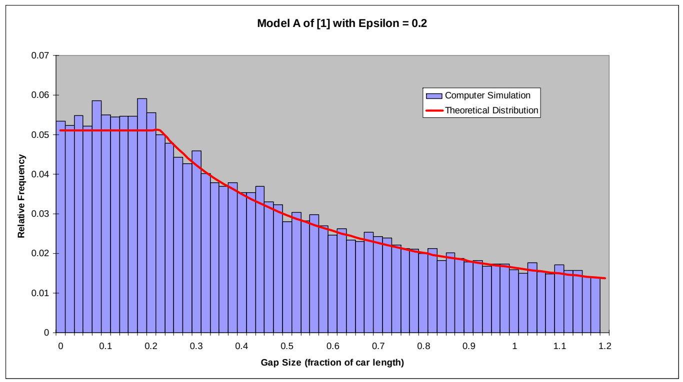
Equations used to derive and model RSA from [3].

Team Members:

Matt Behrens
Anita Lee
Matt Levin
Michael Winslow

Methodology

1. Because RSA alone is insufficient to model car parking, we reproduced the Model A from [1] which incorporates maneuvering room.
2. Model A of [1] failed to fit empirical data, so we reproduced Model B from [1] that incorporates a probability that a car will pull forward after parking.
3. We created a computer simulation to generate additional numerical data to compare to empirical data and gap distributions generated by models A and B of [1].
4. We collected over 250 points of empirical data to compare with models A and B from [1], and also our computer simulation.



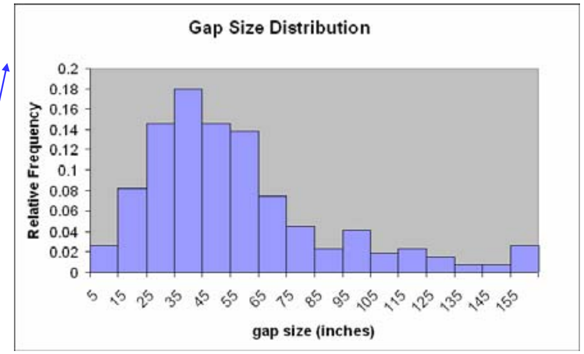
The computer simulation of modified RSA with a maneuvering room of 0.2 with a superimposed graph of Model A using equations from [1]. The superimposed graph was plotted using [4].

Acknowledgments

This project was mentored by Richard Cangelosi, whose help is acknowledged with great appreciation.
Support from a University of Arizona TRIF (Technology Research Initiative Fund) grant to J. Lega is also gratefully acknowledged.

Results

1. Both computer simulated data and empirical data plotted as relative frequency corresponds to [1].
2. Model A [1] did not correctly model gap size distribution of parked cars.
3. Model B, which incorporates cars pulling forward after selecting a spot correctly models the empirical data.



Empirical data collected from the streets of Tucson, AZ.

Glossary of Technical Terms

RSA: Random Sequential Adsorption: process by which particles are randomly adsorbed in a sequential fashion on an infinite line with no overlap.
Adsorption: The adhesion in an extremely thin layer of molecules to the surface of a solid body with which it is in contact.

References

1. Rawal, S. and Rodgers, G.J., *Modelling the gap size distribution of parked cars*, Physica A. **346**, 621-630 (2005).
2. R.J. Field and R.M. Noyes, *Oscillations in chemical systems. IV. Limit cycle behavior in a model of a real chemical reaction*, J. Chem. Phys. **60**, 1877-1884 (1974).
3. Talbot, J., Tarjus, G., Van Tasse, P.R.I, and Viot P., *From car parking to protein adsorption: an overview of sequential adsorption processes*, Colloids and Surfaces A. **165**, 287-324 (2000).
4. Maple(www.maplesoft.com) is developed by Maplesoft, a division of Waterloo Maple, Inc, Waterloo, Ontario, Canada.

RPO: Rotational Position Optimization

Project Description

- Hard Drives are one of the slowest components of computer systems.
- Hard Drives process information as subsequent requests sent by the operating system. A request is sent and will be processed depending on the order in which it was received. As multiple requests bombard the hard drive, a line of requests, or a queue is formed.
- The average delay time associated with the hard drive consists of the seek latency, rotational latency, and the queue delay.
- Since the seek latency and rotational latency are mechanical constraints that cannot be reduced any further we must optimize the queue delay.
- We need a model to help gauge a drive's effectiveness which will mimic the algorithm designed to minimize queue delay implemented by Seagate.
- We cannot however look at the actual algorithm because it is proprietary to Seagate. Thus we must formulate our own model to represent the algorithm. This algorithm is known as RPO or Rotational Position Optimization.

Scientific Challenges

- The RPO algorithm is an interesting problem, it addresses an active area of research investigating technology that will allow a computer to run faster and be more efficient. A model has not yet been developed for the RPO algorithm.

Potential Applications

- We will be able to use this model to analyze the effectiveness of certain hard drives based on their types of applications.

Figure 1: an example of a Hard Drive's standard internal components.

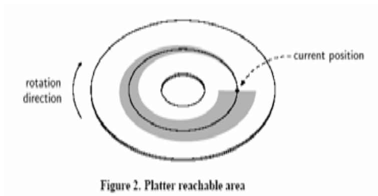


Figure 2: the Reachable area on a platter given a particular start position.

Team Members:

Daniel Norwood
Joseph Ortiz
Chris Summitt
Dr. Olga Yiparaki

Methodology

1. Built a relationship with our representative from IBM and actually research our problem.
2. Acquired Data from IBM DS8000 systems.
3. Analyzed data from two different types of cases, single stream and multi-stream, and formulated models which described the average delay time for both cases. In the case of the single stream, the data was processed in the order it was received, while in the multi stream case, the RPO algorithm was implemented.

Results

1. We formulated a theoretical model using the Seagate manufacturing specifications to predict the average time delay. For the case were the data was sent in at a single stream which didn't implement the RPO, we obtained the following expression.

$$\frac{1}{2}(rotational\_time)+\frac{1}{3}(seek\_time)+data\_transfer+queuing\_delay$$

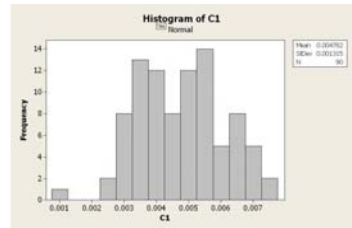
2. Using the data from the multi-stream case, as well as the theoretical model which predicted the average delay for a hard drive not implementing RPO, we constructed a model to predict the delay on a hard drive with RPO enabled. The queuing depth for the specific hard drives was 20, using this as a limit, our theoretical model transformed into the following model which takes into account RPO. The model with no queue is the theoretical model we formulated earlier.
3. We split our model to handle each case, i.e., the case with one access from the queue in the RA versus the case with two accesses in the RA. This gives us:

$$(1-v)\left(\frac{L}{2}+\frac{s}{3}+\sum_{i=1}^{Q}\binom{Q}{i}(1-p)^{Q-i}p^{i}\frac{L}{1+i}\right)+vf$$

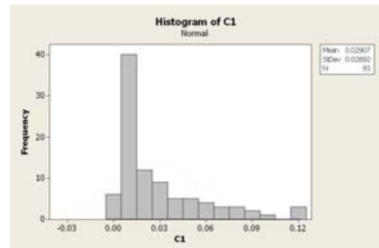
4. Where R is the outer radius of the disk, ρ is the inner radius, Q is the queue depth, L is rotational latency, s is seek latency, and f is a cutoff value to stop starvation.
5. We have found that the values of Q=20, R=3.5, ρ=.5, L=4, s=3.8, f=13 fit our model. The first 5 values came from hardware specifications, the f value is our best estimation.

Glossary of Technical Terms

Seek Latency: time to position heads over cylinder
Rotational Latency: additional time for platters to rotate so that the request is under the head
Queue Delay: Delay caused by the formation of a line when multiple requests for data are sent to the hard drive.
Starvation: When seeks are arranged such that one is never serviced due to its location on the drive.



Distribution for single stream data



Distribution for multi-stream with RPO.

References

1. Seagate Technology LLC, *Economics of Capacity and Speed:Choosing the most effective disc drive size and and RPM to meet IT requirements,* Seagate Global Product Marketing. (May 2004).
2. Bruce L. Worthington, et al, *Scheduling Algorithms for Modern Disk Drives,* Proceedings of the ACM Sigmetrics Conference. (1994).
3. Walter A. Burkhard, John D. Palmer, Rotational Position Optimization (RPO) Disk Scheduling, First Conference on File and Storage Technologies (2002).

Acknowledgments

This project was mentored by Olga Yiparaki, whose help is acknowledged with great appreciation.
Support from a University of Arizona TRIF (Technology Research Initiative Fund) grant to J. Lega is also gratefully acknowledged.

Traffic Flow

Project Description

- To understand the inner workings of macroscopic traffic flow model with the presence of on-ramps.
- To identify phases of macroscopic model to derive origin of equations.
- To understand how dynamic changes in traffic environments due to increase of flux and density.
- To comprehend equations and phase diagrams that are used in previous traffic models (ex. Continuity and Navier-Stokes equations). [2]

Scientific Challenges

- To develop accurate estimations for real world traffic problems

Potential Applications

- To develop new routes, ease congestion, minimize accidents and optimize flow on highways

Equations used in Traffic models

Continuity & Navier-Stokes Equations

(1) $$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0$$

(2) $$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = q_{in}(t)\varphi(x)$$

(3) $$\rho[\frac{\partial v}{\partial t} + v(\frac{\partial v}{\partial x})] = \frac{\partial}{\partial x}(\mu \frac{\partial v}{\partial x}) - \frac{\partial P}{\partial x} + \chi$$

Reduced Navier-Stokes Equation

(4) $$\mu \frac{dw}{dx} = q\left(1 - \frac{C_0{}^2}{v^2}\right)w - \frac{q}{\tau v}\left(V\left(\frac{q}{v}\right) - v\right)$$

(5) $$\frac{dv}{dx} = w$$

Elementary Dimensions		
$[v] = L/T$	$[x] = L$	$[t] = T$
$[\varphi(x)] = 1/L$	$[\rho] = cars/L$	$[q_{in}] = cars/T$
$[\mu] =$ dimensionless	$[P] = [v]^2 * [\rho]$	$[\chi] = cars/T^2$
$\left[\frac{\partial \rho}{\partial t}\right] = \frac{cars}{L*T}$	$[q_{in}\varphi(x)] = \frac{cars}{L*T}$	$\left[\frac{\partial \rho v}{\partial x}\right] = \frac{cars}{L*T}$

Table 1: Dimensions of the variables and parameters used in the traffic flow model

Team Members:

Khoa Han- Electrical Engineering and Mathematics
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Methodology

1. The first traffic model was developed in 1950's by James Lighthill and Gerald Whitham [1].
2. The macroscopic model was examined after careful research of traffic flow models.
3. This continuity equation was analyzed taking into account the presence of a source.
   - In the absence of an on-ramp the external flux, $q_{in}(t)$, through the on-ramp is equal to zero. In other words, the highway will be continuous (i.e., there are no on-ramps or off-ramps).
   - In the presence of an on-ramp the external flux is now included in the analysis (2). $\varphi(x)$ describes the spatial distribution of cars and is a Gaussian distribution with a mean of zero [2].
4. The Navier-Stokes equation (3) for one dimensional pipe flow was used to model traffic flow.
   - Consider water flowing through a pipe. The water enters and exits, if a branch exists, there will be an influx of fluid mixing into the system. The fluid in the main branch will then have to adjust for the added density. The same principle applies to traffic flow on a highway, which is why the authors of [2] used Navier-Stokes equation in order to analyze the highway system.
5. The Navier-Stokes equation was reduced into two systems of differential equations. Data was taken from [2] and plotted in an ODE Simulator (PPLANE [4]). A plot of velocity vs. acceleration is displayed in figure 1.
6. The dimensions of the variables and parameters used in the model are shown in table 1. (Note: μ is the viscosity of the system, χ is the sum of internal forces, and P is pressure)

Simulation Summary

1. Vehicle behavior is displayed when the flux of vehicles entering from the on-ramp is below the stability line (highlighted in black) leading out of point 1 (P1).
2. When the density of vehicles in the system is very high, acceleration spirals downward to zero while velocity approaches the average velocity. This is given as 54 km/hr.

Numerical Simulation of (4) and (5)

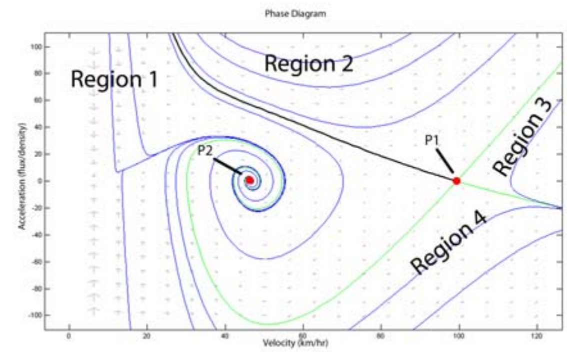

Figure 1: velocity vs. acceleration

Analysis of Regions

Region 1: Vehicles accelerate to average velocity (P2).

Region 2: Density of vehicles is low so vehicles accelerate away from average velocity, increasing with time.

Region 3: Unphysical scenario corresponding to simultaneous existence of high density and high velocity.

Region 4: Vehicles moving at high velocities decelerate to P2.

References

1. Boris S Kerner, *The Physics of traffic*, 25– 30 (1999).
2. H.Y. Lee, H.W. Lee and D. Kim, *Dynamic states of a continuum traffic equation with on ramp*, Phys. Rev. **E 59**. 5101-5111 (1999).
3. Robert W. Fox, Alan R. McDonald, Philip J. Pritchard. *Introduction to Fluid Mechanics*. John Wiley & Sons. United States of America: 2004.
4. PPLANE (http://math.rice.edu/~dfield/) is developed by John C. Polking, Department of Mathematics, Rice University.

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