Short Presentations and Talks at the Arizona School 2012

- Keshav Acharya

  Title: Spectral Theory of a Canonical System

  Abstract: The aim of this presentation is to show that, in the limit circle case, the deficiency index of a Symmetric relation induced by a Canonical System $Ju' = zHu$, is constant on $\mathbb{C}$. This provides a simple proof of the limit point case for the Canonical System when trace $H \equiv 1$. To this end, we first discuss the deficiency indices and spectral theory of any symmetric relation in a Hilbert space $\mathcal{H}$. Then we analyze the spectrum of the relation induced by the Canonical System.

- Michael Aizenman

  Title: Does the 2D Random Field Ising Model exhibit a phase transition in the disorder parameter?

  Abstract: It is a know theorem (which will be explained in the lecture) that in low dimensional models of statistical mechanics, such as the Random Field Ising Model (RFIM), first order phase transitions are unstable with respect to the introduction of arbitrarily weak disorder in the field conjugate to the order parameter. The talk will discuss the possibility that nevertheless there is a phase transition in the model's behavior. Its manifestation could be power law decay of correlations at weak disorder, which at high disorder changes to exponential decay. We shall focus on the model's ground state (i.e. zero temperature). It is proposed that a good point of reference for the scaling limit at weak disorder is Mandelbrot's percolation model. (Joint work with Jack Hanson)

- Sven Bachmann

  Title: On gapped quantum phases and their classification

  Abstract: Despite being widely used, the very notion of a 'ground state phase' has until recently not been well understood. I will argue that the concept of automorphic equivalence of states provides a general scheme and show its relation to other accepted approaches. Furthermore, I will briefly introduce an explicit family of simple models that allow for a classification of gapped phases in one dimension and place the AKLT model within that classification.

- Michael Bishop

  Title: Ground State Energy of the Discrete Random Schroedinger Operator with Bernoulli Potential

  In this talk, I will talk about a recent result showing the probabilistic behavior of the ground state energy in an environment with Bernoulli-distributed potentials. While the distribution of energy can be classified for a wide range of potentials, the Bernoulli-distributed potentials provide special difficulty because they lack density. In our result, this turns out to be a simplifying factor because high potential acts essentially like an infinite barrier for the ground state energy. Experimental work by J. J. Zirbel, S. S. Kondov, W. R. McGehee and B. DeMarco in Science 334, 66, 2011 and J. Billy et al. in Nature 453, 891 behave similarly to the model presented in this talk. I will also share some conjectures for excited states as well as for interacting systems.

Reference: http://arxiv.org/abs/1109.4109 (joint with J. Wehr)

- Zhenwei Cao

  Title: Weak localization for the alloy-type 3D Anderson model

  Abstract: Anderson localization in Lifshitz tails regime is shown for alloy type random Schrodinger operators on a cubic lattice whose randomness is generated by the sign-indefinite single-site potential. A Feynman diagrammatic expansion technique is used on Green's function to provide the initial volume estimate for multiscale analysis. A bound on the integrated density of states is also derived for the positive values of E.

  Reference: Weak localization for the alloy-type anderson model on a cubic lattice. Z. Cao and A. Elgart, Submitted. http://arxiv.org/abs/1108.0196

- Joe Chen

  Title: Bose-Einstein condensation on fractal spaces

  Abstract: I will report on the existence and properties of Bose-Einstein condensate in fractal spaces, most notably on generalized Sierpinski carpets. The main ideas of the proof involve a sharp estimate of heat kernel trace, and the resultant meromorphic spectral zeta function. If time permits I will discuss extensions to interacting Bose gas via development of Gaussian free fields on fractals.

  Reference: http://arxiv.org/abs/1202.1274

- Michael Fauser

  Title: Fractional moment localization in a system of interacting particles in an alloy-type random potential

  Abstract: We discuss results on localization for a system of interacting quantum particles subject to an external alloy-type random potential. Under certain assumptions, there is an energy regime of localization at the bottom of the spectrum of the associated random Schrdinger operator. The proof is an induction on the number of particles and relies on the analysis of fractional moments of the resolvent and their relation to eigenfunction correlators.

- Yulia Karpeshina

Title: Multiscale Analysis in Momentum Space for Quasi-periodic Potential in Dimension Two.

Abstract: We consider a polyharmonic operator $H = (-\Delta)^l + V(x)$ in dimension two with $l \geq 2$, l being an integer, and a quasi-periodic potential $V(x)$. We prove that the spectrum of H contains a semiaxis and there is a family of generalized eigenfunctions at every point of this semiaxis with the following properties. First, the eigenfunctions are close to plane waves $e^{i\langle \vec{k}, \vec{x}\rangle}$ at the high energy region. Second, the isoenergetic curves in the space of momenta $\vec{k}$ corresponding to these eigenfunctions have a form of slightly distorted circles with holes (Cantor type structure). A new method of multiscale analysis in the momentum space is developed to prove these results. This is joint work with Roman Shterenberg.

- Wenbo Li

  Title: Small value probabilities in analysis and mathematical physics

  Abstract: Small value probabilities or small deviations study the decay probability that positive random variables behave near zero. In particular, small ball probabilities provide the asymptotic behavior of the probability measure inside a ball as the radius of the ball tends to zero. In this talk, we will provide an overview with connections and recent developments in analysis and mathematical physics, including metric entropy of compact operators, weaker Gaussian correlation inequality, small ball inequalities, symmetrization inequalities in high dimension, and Laplace asymptotics of partition functions.

- Milivoje Lukic

  Title: Schrodinger operators with decaying oscillatory potentials

  Abstract: We consider a class of Schrodinger operators with oscillatory potentials obeying an L^p decay condition. Our class of potentials includes slowly decaying Wigner-von Neumann type potentials $\sin(ax)/x^b$ with $b > 0$. We prove absence of singular continuous spectrum and show that embedded eigenvalues in the continuous spectrum can only take values from an explicit finite set. Conversely, we construct examples where such embedded eigenvalues are present.

  The preprint is available at arXiv:1201.4840.

- Rajinder Mavi

  Title: Quasiperiodic Schrodinger operators with rough potentials

  Abstract: Many complete results are known for the 1-dimensional quasiperiodic Schrodinger operator with potential defined by analytic sampling functions, but the situation for sampling functions with less regularity is relatively unknown. We discuss methods and extensions of results to the case of Lipshitz continuous sampling functions.

- Clark Musselman

  Title: Diffusion for a Markov, Divergence-form Generator

  Abstract: We consider the long-time evolution of solutions to a Schrödinger-type wave equation on a lattice with a Markov random generator. We show that solutions to this problem possess a diffusive scaling limit. We also compute higher moments.

  Based on joint work in preparation with Jeffrey Schenker.

- David Renfrew

  Title: Fluctuations of outlier eigenvalues in deformed Random Wigner Matrices

  Abstract: In this talk, I will discuss the fluctuations of outlier eigenvalues in deformed Wigner Matrices. This research extends the work Capitaine, Donati-Martin, and Féral studying the eigenvalues of $W_N + A_N$ where W_N is a Wigner matrix and A_N is a finite rank deformation. We study Wigner matrices with only four finite moments and no symmetry restriction on the entry's distribution. In particular we show that both the fluctuations and expectation of the outlier eigenvalues is dependent on the form of the deformation.

  Reference: http://arxiv.org/abs/1103.3731, accepted for publication in Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques.

- Christian Sadel

  Title: Absolutely continuous spectrum for random Schrdinger operators on tree-strips of finite cone type.

  Abstract: A tree-strip of finite cone type is the product of a finite graph with a tree of finite cone type. We consider random operators on these tree strips and prove that for small disorder, the spectrum is purely absolutely continuous in a certain set.

- Jeffrey Schenker

  Title: Diffusion of waves in a random environment: problems and results

  Abstract: I will discuss the "quantum diffusion" conjecture, namely that solutions to the lattice Schrödinger equation exhibit diffusive motion transport over long time and space scales if the disorder is weak enough (in three or more spatial dimensions). A major difficulty that arises when considering this problem is recurrence – return of portions of the wave packet to regions previously visited. I will show some recent results that show, if recurrence is eliminated by making the environment evolve randomly in time then diffusion results in an elementary way.

- Christian Seifert

  Title: A Gordon type theorem for measure perturbed Schrdinger operators

  Abstract: We prove a version of Gordon's Theorem for measure perturbed one-dimensional Schrdinger operators and show absence of eigenvalues for these operators.

  Reference: C. Seifert, Gordon type theorem for measure perturbation, EJDE 2011, No. 111.

- Brian Simanek

  Title: Ratio and relative asymptotics of general orthogonal polynomials

  Abstract: I will present some new results concerning ratio asymptotics and relative asymptotics of orthogonal polynomials whose measure of orthogonality is supported on an arbitrary compact subset of the complex plane. In this setting, there is no finite term recursion relation among the orthonormal polynomials, so one needs to use new techniques to study these polynomials.

- Shannon Starr

  Title: Bounds on fluctuations for Mallows random permutations

  Abstract: Instead of considering a uniform random permutation in S_n consider a probability measure that assigns π the probability proportional to q-to-the-power of the number of inversions of π: an inversion of π is a pair i, j with $i < j$ and $\pi(i) > \pi(j)$. When $q = 1$ this is the uniform measure still. Taking q close to 1, more precisely $\exp(-\beta/n)$ for some real β, results in a sequence of measure "close to uniform" as n goes to infinity. We consider the fluctuations of the length of the longest increasing subsequence. While they should probably be $n^{1/6}$ according to some form of universality or a cross-over distribution, we use Talagrand's inequality to bound them by $n^{0.25+}$ along suitable subsequences. We also discuss the heuristic cavity step. This is joint work with Meg Walters, also from UR.

- Scott Strong

  Title: Generalized Local Induction, Hasimoto's Map and Admissible Vortex Geometries

  Abstract: Turbulence is an unsteady flow where diffusion, dissipation and vorticity occur on various space-time scales with vorticity acting as an internal skeleton determining the structure of a fluid flow. Quantum effects constrain aspects of turbulence making the superfluid case, in some sense, simpler than its classical counterpart. For such a superfluid flow the vorticity must manifest as quantized vortex filaments[1] and the tangling of these filaments is the hallmark of quantum turbulence.[2] The total concentration of vorticity to a plane-curve is particularly appropriate for atomic Bose-Einstein condensates and superfluid Helium and allows for aggressive analytic and numerical investigations. Specifically, recent work exploits this geometric constraint and, without approximation, provides an analytic representation of the induced velocity field in terms of elliptic integrals whose asymptotic representation generalizes the established local induction approximation predicting binormal filament flow.[3,4] Interestingly, in 1972 Hasimoto showed that such filament flows can be mapped onto the focusing cubic nonlinear Schrdinger equation (NLS).[5] Thus, solutions to NLS can, in theory, be used to prescribe admissible geometries of vortex filaments. However, the problem of recovering the filament geometry from the Serret-Frenet equations (SF), which is a non-autonomous system of linear ODEs, is nontrivial. While it is possible to numerically approximate solutions to SF to recover the filament geometry, this tells us nothing about the properties of the inversion of Hasimotos mapping. However, it is possible to reduce the SF problem to a simpler 2×2 problem through the use of the famous SU (2) to SO(3) map. From this one can study the NLS to vortex filament mapping and easily visualize possible geometries.[6] This talk will discuss the recent Generalized Induction Equation [7] and the inversion of Hasimoto's famous map.

[1] R. J. Donnelly, Quantized Vortices in Hellium II, Cambridge University Press, (2005).

[2] W. F. Vinen and J.J. Niemela, Quantum Turbulence, J. Low Temp. Phys., 128, (2002), 167-231.

[3] F. R. Hama, Genesis of LIA, Fluid Dynamics Research, 3, (1988), 149-150.

[4] R. L. Ricca, The contributions of Da Rios and Levi-Civita to asymptotic potential theory and vortex filament dynamics, Fluid Dynamics Research, 18, (1996), 245-268.

[5] H. Hasimoto. A soliton on a vortex filament. Journal of Fluid Mechanics, 51:477485, 1972.

[6] P. G. Grinevich and M. U. Schmidt. Closed curves in $\mathbb{R}^3$: a characterization in terms of curvature and torsion, the Hasimoto map and periodic solutions of the Filament Equation. In eprint arXiv:dg-ga/9703020, page 3020, March 1997.

[7] S. A. Strong and L. D. Carr, Generalized Local Induction Equation, Elliptic Asymptotics and Simulating Superfluid Turbulence, J. Math. Phys., under review (2011). arXiv:1102.2258v1

- Nikolai Veniaminov

  Title: Existence of Thermodynamic Limit for Interacting Quantum Particles in Random Media.

  Abstract: A system of quantum particles interacting through a pair potential and subject to a random background potential is studied. It is proved that the

energy and the entropy of such system grow approximately linearly when the number of particles and the volume of the system go to infinity jointly in a way that the particle density is kept constant. The proofs are mainly based on subadditive type inequalities.

Reference: http://arxiv.org/abs/1112.2575

- Anna Vershynina

  Title: Non-equilibrium state of a leaking photon cavity pumped by a random atomic beam.

  Abstract: We consider a system that consists of an atomic beam shooting one atom at a time and a cavity, which atoms enter consequently. First we assume that the cavity insulates the atom-photon system from decohering interactions with its environment. In this case we show that the number of photons in the cavity increases linearly in time and only flux of randomly exited atoms is able to produce a pumping of the cavity by photons. As a more general case we allowed the photons to leak out of the cavity at some constant nonzero rate, so that the evolution of the system is described by a Markovian dynamics. We showed the asymptotic behavior of the limiting state and proved that the state is non-equilibrium. Also we proved that the number of photons in the cavity stabilizes in time in this case.