MATH 129-020
SOME WORD PROBLEMS

SPRING 2019

(1) Let $Q(t)$ be a quantity that changes over time t. Suppose $Q(t)$ satisfies the following differential equation:

$$\frac{dQ}{dt} = kQ \quad \text{where } k \text{ is some real-valued constant.}$$

a) Solve this differential equation for the initial condition $Q(0) = Q_0$.

b) Suppose $k > 0$. Find the doubling time for $Q(t)$.

c) Suppose $k < 0$. Find the 1/2-life for $Q(t)$.

(2) A bank account earns interest that is continuously compounded at a rate of 5% of the current balance per year. What is the corresponding annual growth rate?

(3) If we know that a certain population is growing at a rate of 2% per year, find the continuous growth rate.

(4) An egg, initially at $15°C$, is put in a pot of boiling water (at $100°C$).

a) Write an initial-value problem which describes the temperature $T(t)$ (in $°C$) of the egg as a function of time t (in minutes) after the egg is placed in the boiling pot.

b) Find the solution of the differential equation in part a).

c) Suppose that after 1 minute the egg is $35°C$. At what time will the egg reach $90°C$? Give an exact answer and then approximate this value with two decimal place accuracy.

(5) When a murder is committed, detectives often use Newton's Law of Cooling to approximate time of death. This problem illustrates how this is done.
Suppose a body is found in a room. Suppose that the room's temperature is always kept at 20°C. Recall that $37°C$ is the average human body temperature.

a) Find the temperature of the body $T(t)$ as a function of time T in hours after the murder; i.e. declare that $t = 0$ corresponds to the time of the murder.

b) If the body is found at 2 PM and its temperature is measured to be $35°C$ and then at 4 PM the temperature is found to be $30°$, find the time of the murder. Assume the body has not been moved.