MATH 396T
TEST 1
SIMS

SPRING 2020

Name	
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total	40	

Directions: Show all work. You may use any result proven in class, or homework problem, but you must state the results you are using precisely.

(1) a) Let $f : \mathbb{R} \to \mathbb{C}$ be a function whose real and imaginary parts are differentiable. Let $g : \mathbb{R} \to \mathbb{R}$ also be a differentiable function. Show that the chain rule holds, i.e. let $h : \mathbb{R} \to \mathbb{C}$ be the function $h(x) = (f \circ g)(x) = f(g(x))$, and show that

$$h'(x) = f'(g(x))g'(x) \quad \text{for all } x \in \mathbb{R}\,.$$

b) Use the above to calculate the derivative of

$$f(x) = e^{2i\cos(x)}\,.$$

(2) Consider the complex number

$$z = e^{3-2i}$$

Let $w = e^z$.

a) Find $\mathrm{Re}[w]$.

b) Find $\mathrm{Im}[w]$.

c) Find $|w|$.

(3) For any arithmetic function f with period $q \in \mathbb{N}$, define

$$\|f\|_1 = \sum_{n=1}^{q} |f(n)|.$$

Show that if f and g are arithmetic functions, both with period $q \in \mathbb{N}$, then

$$\|f * g\|_1 \leq \|f\|_1 \|g\|_1$$

Here $f * g$ is the convolution of f and g.

(4) Let $a \in \mathbb{C}$. Consider the arithmetic function f with period 4 satisfying

$$f(0) = -1, \quad f(1) = a, \quad f(2) = 1, \quad \text{and} \quad f(3) = -a\,.$$

i) Evaluate all four values of $e(-\frac{n}{4})$ with $n = 0, 1, 2, 3$. (Recall that our convention is that $e(x) = e^{2\pi i x}$ for all $x \in \mathbb{R}$.)

ii) Find the Discrete Fourier Transform of the function f described above, i.e. find $\hat{f}(k)$ for all $k \in \mathbb{Z}$. Use your results in part i) to simplify these expressions.