# MATH 425A
# EXAM 1

FALL 2019

Show all work. You may use any result proven in class, or homework problem, but you must state the results you are using precisely.

(1) Let $x_1, x_2, y_1, y_2 \in \mathbb{R}$. Show that if $x_1 \leq y_1$ and $x_2 \leq y_2$, then

$$x_1 + x_2 \leq y_1 + y_2$$

Show also that if one of the assumed inequalities is strict, then the conclusion holds with a strict inequality.

(2) Use induction to prove the following: Let $a \geq 0$. For any $n \in \mathbb{N}$, show that

$$(1+a)^n \geq 1 + na + \frac{n(n-1)}{2}a^2$$

(3) Define a sequence recursively by setting $x_1 = 1$ and then for any $n \in \mathbb{N}$ taking

$$x_{n+1} = \frac{4 + x_n}{5 + x_n}\,.$$

a) Show that $x_n$ is positive for all $n \in \mathbb{N}$.
b) Show that $\{x_n\}$ is monotonically decreasing.
c) Calculate

$$\lim_{n\to\infty} x_n$$

(4) a) Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Suppose that $f(x) = 0$ for all rational real numbers $x$. Prove that $f(x) = 0$ for all real $x$.

b) Let $a > 0$. Suppose $f : [-a, a] \to \mathbb{R}$ is continuous. Suppose further that

$$f(-a) > -a \quad \text{and} \quad f(a) < a\,.$$

Show that there is $x \in (-a, a)$ for which $f(x) = x$.