MATH 425A:
EXAM 1
MAKE-UP

FALL 2019

Name	
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total	40	

Directions: This work is an optional assignment for those who took the first test on Friday, October 4th. It is due on Monday, October 14th at the beginning of class. No late work will be accepted. If you turn this in, I will grade it and your new grade on Exam 1 will be the average (out of 100%) of the two scores you have received. If you do not turn this in, your grade on Exam 1 will stay the same.

Show all work. You may use any result proven in class, or homework problem, but you must state the results you are using precisely.

(1) Let $n \in \mathbb{N}$. Suppose $x_j \geq 0$ for all $1 \leq j \leq n$. Prove that

$$\sum_{j=1}^{n} x_j \geq 0\,.$$

In addition, show that: if $n \in \mathbb{N}$ and $x_j \geq 0$ for all $1 \leq j \leq n$, then

$$\sum_{j=1}^{n} x_j = 0 \quad \text{if and only if} \quad x_j = 0 \quad \text{for all } 1 \leq j \leq n.$$

(2) a) Consider the sequence $\{a_n\}$ with terms given by

$$a_n = \frac{1}{\sqrt{n+3}} + \frac{n-3n^2}{2n^2-1} \quad \text{for all } n \in \mathbb{N}\,.$$

Use the Archimedean Property to demonstrate that $\{a_n\}$ converges and determine the value of the limit.

b) Determine whether the sequence $\{x_n\}$ with terms given by

$$x_n = \sqrt{\frac{4n^2-1}{n^2+1}} \quad \text{for all } n \in \mathbb{N}$$

has a limit. If it does, find its limiting value.

(3) Let $f : (0, 1] \to \mathbb{R}$ be a continuous function. Let $\{x_n\}$ be a sequence in $(0, 1]$ for which $\{f(x_n)\}$ is unbounded. Prove that $\{x_n\}$ has a sub-sequence that converges to 0.

(4) Let $n \in \mathbb{N}$. Consider the set

$$\mathcal{D} = \bigcup_{k=1}^{n} [a_k, b_k] \, .$$

In words, $\mathcal{D}$ is the union of n closed, bounded intervals.
a) Show that $\mathcal{D}$ is closed and bounded.
b) Show that $\mathcal{D}$ is sequentially compact.
c) Let $f : \mathcal{D} \to \mathbb{R}$ be continuous. Show that $f : \mathcal{D} \to \mathbb{R}$ is uniformly continuous.
d) Show that if $f : \mathcal{D} \to \mathbb{R}$ is continuous, then the extreme value theorem holds.