INTRODUCTION TO MATHEMATICAL PHYSICS: QUANTUM SPIN SYSTEMS
MATH 541: SECTION 001
SYLLABUS

FALL 2018

Instructor:	**Office:**	**Office Hours:**	**Phone:**
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1. **Brief Course Overview:**

Non-relativistic quantum mechanics describes atoms, molecules, and both small and large systems composed of atoms and molecules. The traditional way in which quantum spin systems arise is by a reduction of the Hilbert space of states for each atom or molecule to a finite-dimensional subspace and such reductions can often be justified on physical grounds. Other ways in which quantum spin models arise are: as a truncation of a lattice quantum field theory for the purpose of numerical simulation, as collections of qubits in quantum information theory, and as toy models in some theories of quantum gravity.

Many interesting features of quantum many-body physics can already be found in quantum spin models. For example, these include the complex dynamics due to interactions between the components (be it particles or spins), the possibility of phase transitions, the important role played by symmetries and spontaneous symmetry breaking, the unique behavior typical of quantum phases of matter such as Bose-Einstein condensation and superfluidity, superconductivity, the integer and fractional quantum Hall effects, topological order, exotic quasi-particles called anyons etc. Quantum spin models provide the simplest framework in which all these phenomena can be studied in detail. It is also the setting that has proved to be most amenable to rigorous mathematical analysis. In fact, research on quantum spin systems has led to significant new development in functional analysis (e.g., the theory of operator algebras) and representation theory (e.g., quantum groups).

With these lectures, I have two main goals. The first is to provide a basic introduction to the mathematical framework for the rigorous study of quantum spin models and to introduce some important models. The second goal is to discuss some important directions of current research on quantum spin models.

2. Textbook:

For this course, I will assume a working knowledge of linear algebra and differential equations. There are many good textbooks reviewing these topics, and I can give any interested students references. My goal is to make the material presented accessible to graduate students of mathematics and physics, but anyone with an interest in the subject matter is welcome to attend.

I plan to mainly use my own notes. Useful further reading may include:

(1) R. Bhatia, *Matrix Analysis*, Springer 1997.
(2) O. Bratteli and D. W. Robinson, *Operator algebras and quantum statistical mechanics*, 2 ed., vol. 1, Springer Verlag, 1987.
(3) O. Bratteli and D. W. Robinson, *Operator algebras and quantum statistical mechanics*, 2 ed., vol. 2, Springer Verlag, 1997.
(4) E. Carlen, *Trace Inequalities and Quantum Entropy: An Introductory Course*, AMS Contemporary Mathematics 529, 2010.
(5) I. Lankham, B. Nachtergaele, and A. Schilling, *Linear Algebra as an Introduction to Abstract Mathematics*, World Scientific, 2016.
(6) P. Naaijkens, *Quantum spin systems on infinite lattices*, Lecture Notes in Physics 933, Springer. See also https://arxiv.org/abs/1311.2717.
(7) D. Petz, *Quantum Information Theory and Quantum Statistics*, Springer 2008.
(8) D. Ruelle, *Statistical Mechanics: Rigorous Results*, World Scientific 1999.

3. Goals:

My goals are to cover most of the following material:

- Quantum Spin Systems - basic structure
- Schrödinger and Heisenberg Dynamics
- Lieb-Robinson Bounds
- Ground States and Thermal States
- Infinite Systems and the GNS Construction
- The Exponential Clustering Theorem
- The Goldstone Theorem
- Models
  - The XXZ Model
  - The XY Model
  - The AKLT Model

4. Grading Policy:

There will be two distinct paths for obtaining your final grade.

First, I will regularly assign homework. If you choose, I will base your course grade on the homework problems you have completed.

Next, there will be course projects. If there is interest, I will give you a topic relevant to the course material. In this case, you will give a presentation on this topic (one class lecture), and your course grade will be based on this.