PARAMETRIZATION OF RINGS OF SMALL RANK: COURSE DESCRIPTION

M. BHARGAVA

A *ring of rank n* is a commutative ring that is free of rank n as a $\mathbb{Z}$-module, i.e., isomorphic to $\mathbb{Z}^n$ as an abelian group. The prototypical example of a ring of rank n is an order in a degree n number field. In this series of lectures, we investigate how rings of small rank n can be explicitly parametrized.

The cases $n = 4$ and $n = 5$ are particularly interesting, as they are connected to the arithmetic theory of quadratic forms – a connection which begs for further investigation!

In the first lecture, we will review the classification of rings of ranks ≤ 3, before turning to the case $n = 4$ (i.e., the case of "quartic rings").

In the second lecture, we will examine the case $n = 5$ (i.e., the case of "quintic rings").

The third lecture will then examine how these various parametrizations can in fact be treated not only over $\mathbb{Z}$, but over an arbitrary base ring.

Finally, in the last lecture we will consider what happens when the "commutative" condition is dropped, leading to explicit parametrizations of "quaternion rings" and "octonion rings". Applications will be discussed.