Let M be a free finite rank $\mathbb{Z}_p$-module.

Let $U \circlearrowright M$ cont. linear op.

Then $e := \lim_{n\to\infty} U^{n!}$ is a projector ($e^2 = e$) of M onto the subspace eM on which U is invertible.

Let M_k^{ord} = p-adic mod forms of weight k. over $\mathbb{Z}_p$.

<u>Thm</u> [Hida] $e_p := \lim_{n\to\infty} U_p^{n!}$ is a projector of M_k^{ord} onto the subspace where U_p is inv. Moreover...

$F \in M_K^{ord}$.

$F = A + B$.

$A = eF \qquad B = (1-e)F$.

$A =$ finite sum of eigenforms.

$U^m B \longrightarrow 0$

converging p-adically to zero.

$U_p^m F \longrightarrow U_p^m A$

1. $\dim e_p M_k^{ord} < \infty$

only depends on $k \bmod p-1$

$2 .(p=2).$

2. $k \geqslant 2$

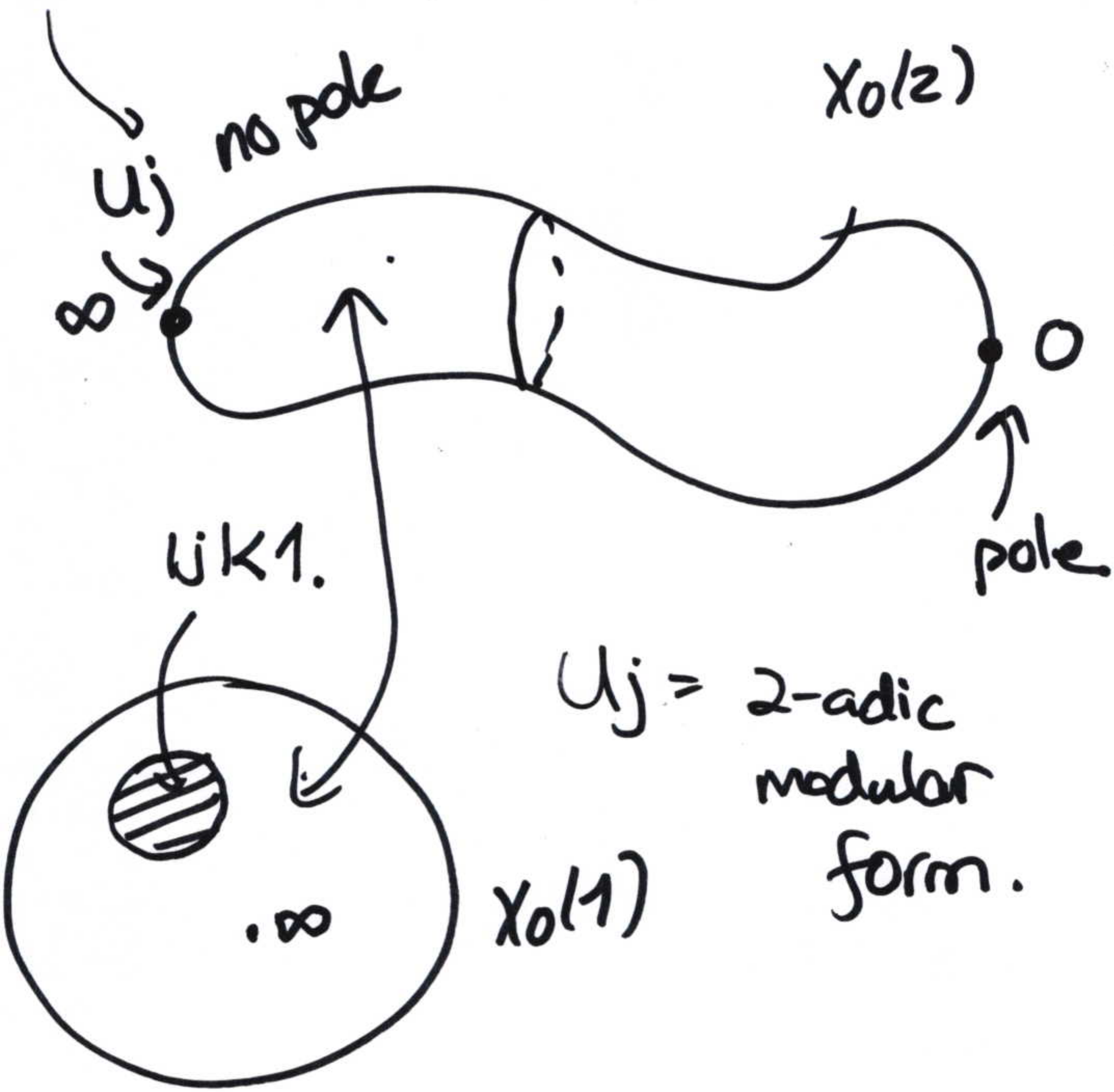
$$e_p M_k^{ord} \subseteq M_k(\Gamma_0(p), \mathbb{Z}_p)$$

can be decomposed into eigenforms for T_ℓ, $\ell \neq p$ and U_p. (for any k).

This implies many congruences!

j : pole at ∞.

U_j = meromorphic function on $X_0(2)$.



U_j = 2-adic modular form.

$$e_2(U_j) \subseteq M_0^+(\Gamma(1), \mathbb{Z}_2)$$

$k=0 \Rightarrow k=2$

$$e_2(M_2^+) \subseteq M_2(\Gamma_0(2), \mathbb{Z}_2)$$

$$e_2(U_j) = 744.$$

$$U_j = 744 + (U_j - 744).$$

$$U^m_j = 744 + n \rightsquigarrow 0.$$

$c(n) \longrightarrow 0$ if $n \longrightarrow 0$ in $\mathbb{Z}_2$ $(n \neq 0)$.

§ Overconvergence.

classical level $\Gamma_0(p)$

$\subseteq$ p-adic modular forms.

E ordinary $\Longrightarrow$ $P \subseteq E[p]$

$p=2$ $K/\mathbb{Q}_2$ $\mathcal{O}=\mathcal{O}_K$ $\mathfrak{m}$.

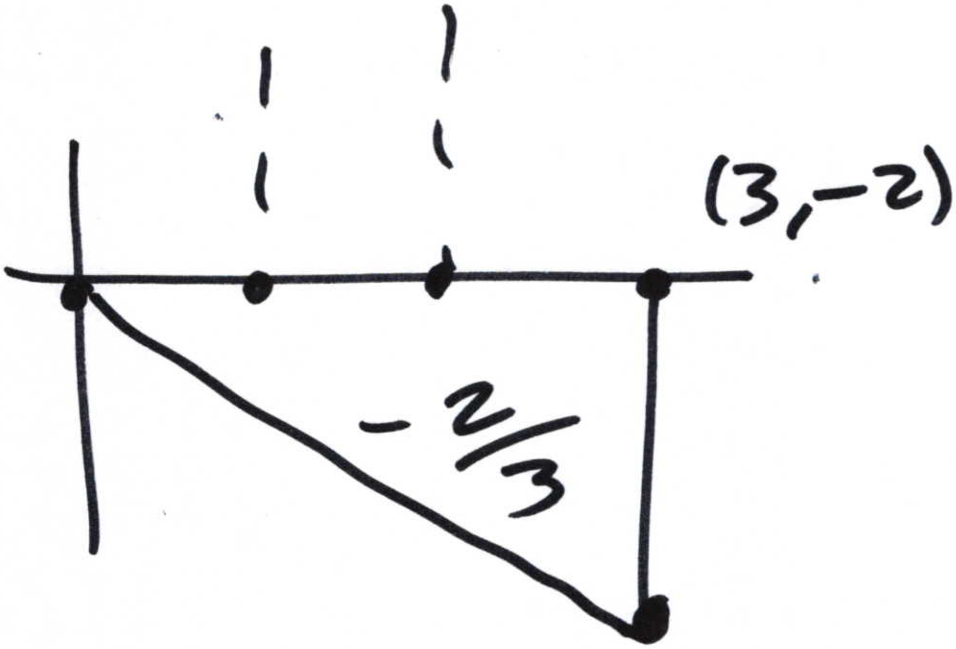
$$y^2+a_1xy+a_3y = x^3+a_2x^2+a_4x+a_6.$$

$a_1 \bmod 2$ = Hasse Invariant.

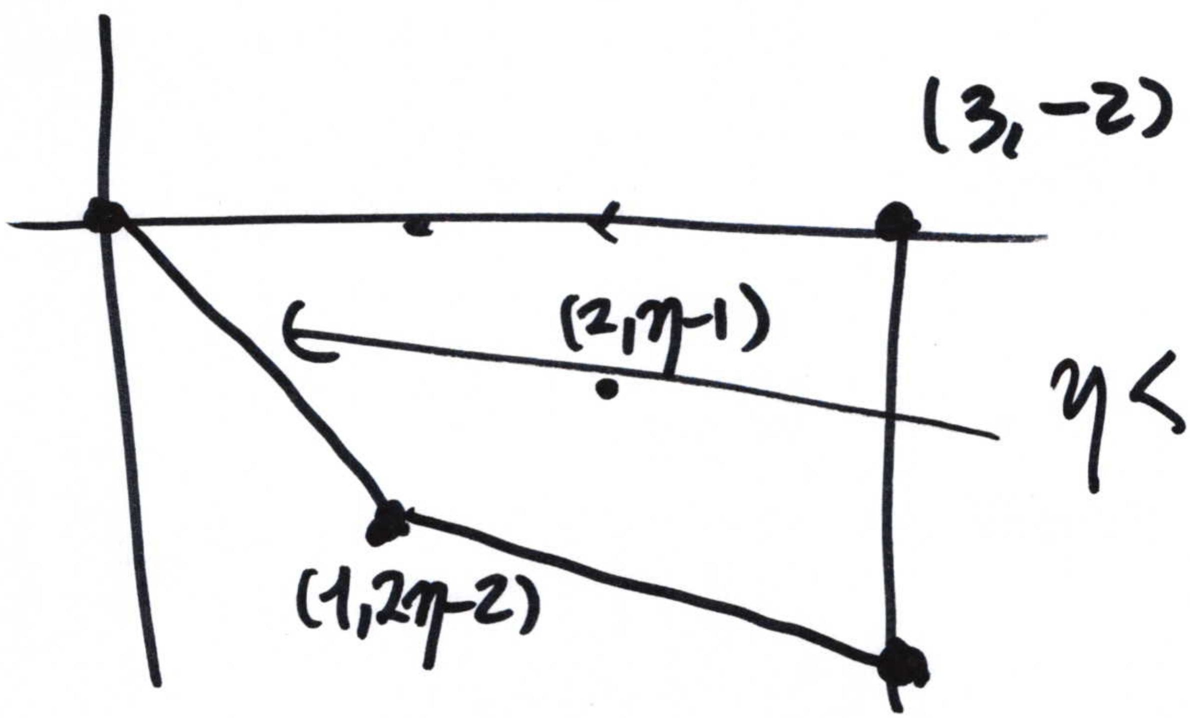
$$\left(y+\frac{a_1x}{2}+\frac{a_3}{2}\right)^2 = x^3+\left(a_2+\frac{a_1^2}{4}\right)x^2 + \left(a_4+\frac{a_1a_3}{2}\right)x + a_6+\frac{a_3^2}{4}.$$

<u>a_3 unit</u>

<u>case 1</u> $v(a_1) \geq 1$.

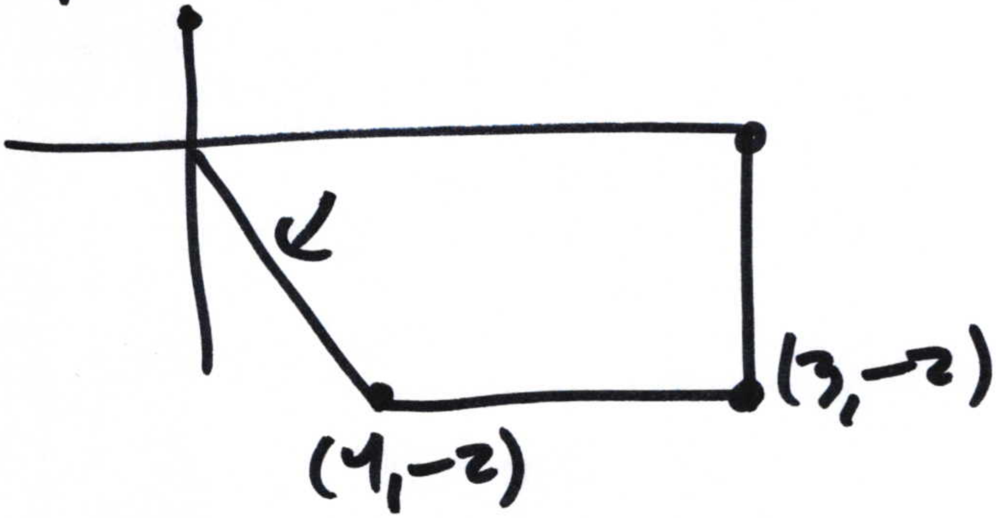


case 2: $1 > v(a_1) > 0$.
$\quad\quad\quad\quad\quad\quad\quad \overset{\shortparallel}{\eta}$



$\eta < \frac{2}{3}$

$\eta = 0$



<u>Thm</u> If $V(A(E,\omega)) < \frac{2}{3}$,

$\exists$ canical $P \subseteq E[2]$.
on

FC4-9

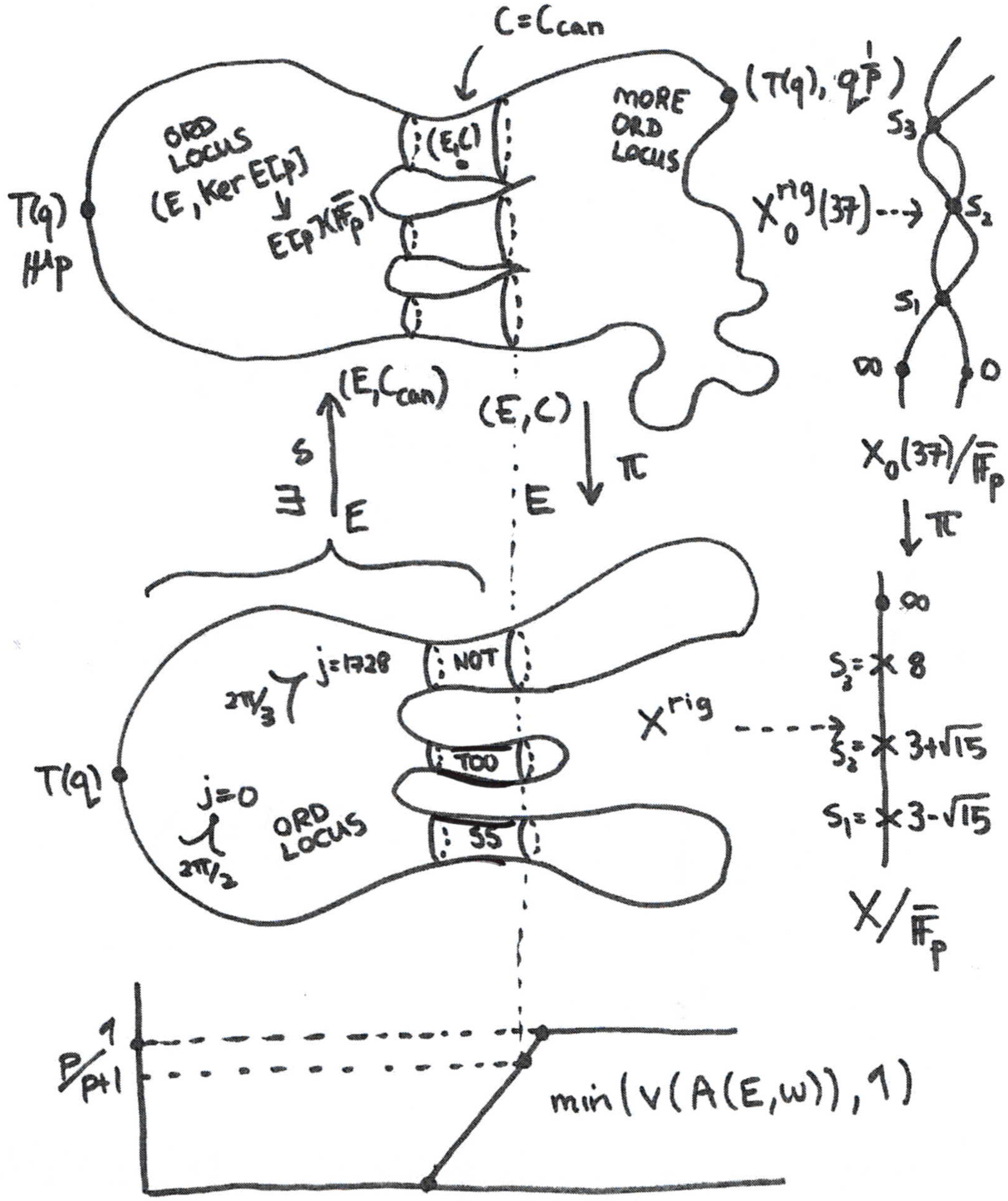


FIGURE 3. The map $X_0^{\mathrm{rig}}(37) \to X^{\mathrm{rig}}$ drawn as if $\mathbf{C}_{37}$ were archimedean

The correct way to think about this is that the operator U_p *increases* the convergence of an overconvergent modular form. The next thing to consider is what type

$M^{\dagger}_{k}(\Gamma, r)$

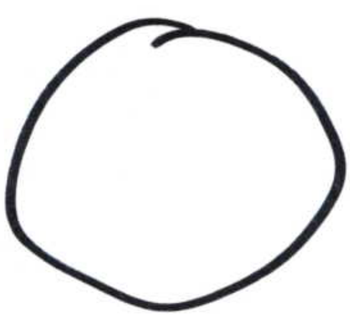
$= H^{0}(X(r), \omega^{\otimes k})$

$(p=2)$:

$V(A(E,\omega)) \leq r.$ $(r<1)$.

$X(0) = X^{ord}$.

$X_0(1)$ $X_0(2)$



$$f = q\prod_{n=1}^{\infty}(1+q^{n})^{24}.$$

2-adic modular forms.

function $|j^{-1}| \leq 1$

$= \mathbb{C}_2[\![j^{-1}]\!] = \sum a_n j^{-n}$

$|a_n| \to 0.$

$\|j^{-1}\| < |p^t| \qquad t > 0$

$(p=2).$

$$\begin{tikzcd}[row sep=small] & P \\ E \arrow[ur, dash] \arrow[r, dash] \arrow[dr, dash] & Q \\ & R \end{tikzcd}$$

of order 2.

$v(A(E,\omega)) = \eta$.

P = can	$\times$	$\times$	
E/p	E/Q	E/R	.
0	0	0	$v(A(\times))$ $\eta = 0$
$p\eta$	$\frac{\eta}{p}$	$\frac{\eta}{p}$.	$\eta \neq 0$.

⋮

$$F \in M_k^{\dagger}(\Gamma, r)$$

$$UF \in M_k^{\dagger}\left(\Gamma, \min\{pr, \tfrac{p}{p+1}\}\right).$$

$$UF(E) = \sum_{P \neq \underline{\text{can}}.} f(E/P)$$

$v(A(E)) \leq pr$ $\qquad$ $v(A(E/P)) \leq r.$

$$\begin{tikzcd}
U: M_k^{\dagger}(\Gamma, r) \arrow[r] \arrow[dr] & M_k^{\dagger}(\Gamma, pr) \arrow[d, dashed, "\text{rest}"] \\
& M_k^{\dagger}(\Gamma, r)
\end{tikzcd}$$

$\therefore$ U is a compact operator.

$C(r) = $ c.anly $\mathrm{rad}(f) \leq r.$

$$C(1) \longrightarrow C(2) \dashrightarrow C(1)$$

$$f(z) \quad f\left(\frac{z}{2}\right) \dashrightarrow f\left(\frac{z}{2}\right)$$

$$\{1, \underbrace{z, z^2, z^3, \ldots\ldots}_{\text{eigenforms.}}\}$$

<u>Hope</u> : compactness of U implies:

$F \in M_k^{\dagger}(\Gamma, r)$

$$F = \sum_{i=1}^{\infty} \alpha_i \phi_i$$

ϕ_i = eigenforms for U_p and T_ℓ, $(\ell \neq p)$

$U\phi_i = \lambda_i \phi_i$

$\lambda_1, \lambda_2, \lambda_3 \dots$

$|\lambda_1| \geq |\lambda_2| \geq |\lambda_3| \geq \dots$

$$U = \begin{pmatrix} 0 & 0 & 0 & 0 & \\ p & 0 & 0 & 0 & \cdot \\ 0 & p^2 & 0 & 0 & \cdots \\ 0 & 0 & p^3 & 0 & \cdots \\ 0 & 0 & 0 & p^4 & \cdots \end{pmatrix}$$

Asymptotic Expansion.

Given $F \in M_k^+(\Gamma, r)$

$$F \sim \sum \alpha_i \phi_i$$

fix $h > 0$

$$U^k\Big(F - \sum_{|\lambda_i| > |p^h|} \alpha_i \phi_i\Big) = o(p^{hk}).$$

$$\frac{1}{\eta} = q^{-\frac{1}{24}} \sum_{n=0}^{\infty} p(n) q^n.$$

$p = 5.$

$$U \frac{1}{\eta} \sim \underline{\underline{\alpha_2 \phi_2}} + \alpha_7 \phi_7 + \alpha_9 \phi_9 + \cdots$$

$$U \phi_k = \lambda_k \phi_\lambda$$

$$|\lambda_k| = |p^k|.$$