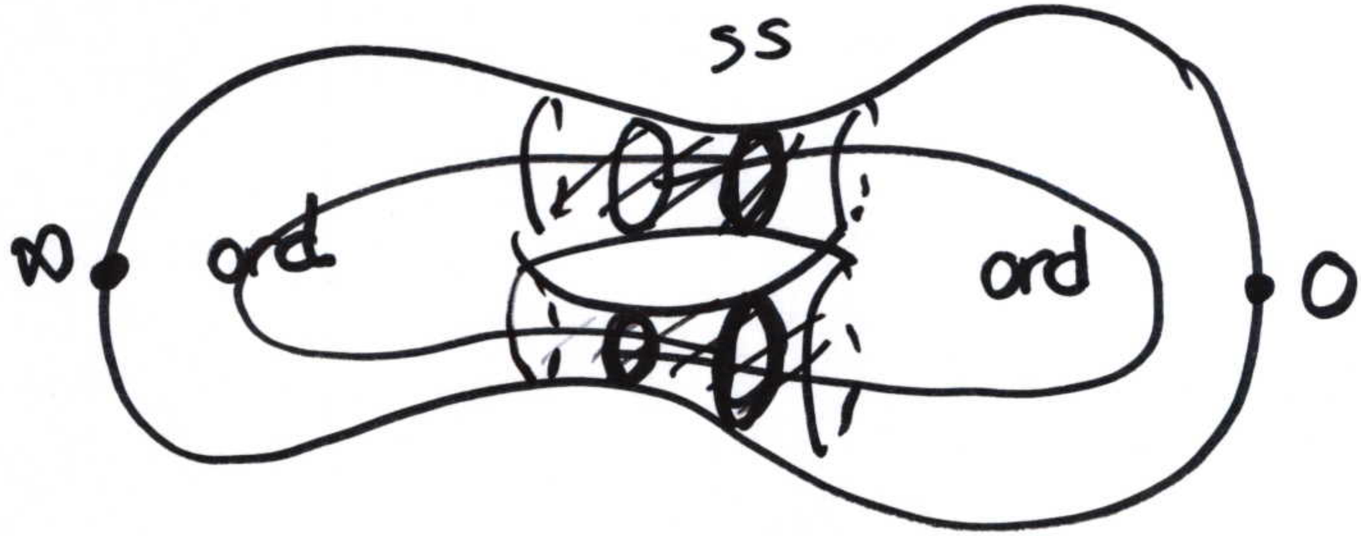
$X_0(p)$



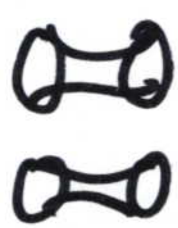
$w: (E, P) \longrightarrow (E/P, E[p]/P)$

$w^2 = 1.$

$r \in \left(\frac{1}{1+p}, \frac{p}{1+p}\right)$

$r \geq \frac{1}{2}$

$M_k^\dagger(\Gamma, r) \cap w M_k^\dagger(\Gamma, r)$

f defined  inside ss.

$r \geqslant \frac{1}{2}$ $\quad k = 0$

$M_0^+(\Gamma, r) \times M_0^+(\Gamma, r)$

$f \quad , \quad wg$

$\langle f, g \rangle := \int wg \, df$

$\mathrm{Res}_\infty \, wg \cdot df$

$\int \sum_{-\infty}^{\infty} a_n t^n \, dt = a_{-1}.$

$\langle\, , \rangle$ is Hecke equivariant.

$$\langle U f, g\rangle = \langle f, U g\rangle$$

$$\langle T_\ell f, g\rangle = \langle f, T_\ell g\rangle.$$

- If $r > \frac{1}{1+p}$

  $\ker U = 0.$

- If $r \geq \frac{1}{2}$

on $M_0^\dagger(\Gamma, r)$ the op.

U is self-adjoint wrt $\langle , \rangle$.

Problem.

$N \in M_{\infty \times \infty}(\mathbb{Z}_p)$ compact

(i) $\ker M = 0$

(ii) M symmetric

$\Rightarrow$ what can you deduce about M?

<u>Congruences.</u>

Given $F \in M_k^+(\Gamma, r)$

$$F = \sum_{i=1}^{\infty} \alpha_i \phi_i$$

eigenforms s/ $\phi_i = \lambda_i \phi_i$.

<u>k=0</u>.

Upgrade of ~~[illegible]~~.

Conj: $$F = \sum_{i=0}^{\infty} \frac{\langle F, \phi_i \rangle}{\langle \phi_i, \phi_i \rangle} \phi_i.$$

$N=1,\ p=2.\ k=0$

theorem of <u>Loeffler</u>.

$$u_j = \underbrace{e_p u_j}_{\text{finite sum of eigenforms.}} \text{ "+" } \sum_{|\lambda_i|<1} \alpha_i \phi_i .$$

$$c(p^n k) = \underbrace{}_{\text{eigenforms}} + O(p^{nc})$$

$$c := \min\{v(\lambda_i) \mid v(\lambda_i) > 0\}.$$

<u>Lemma</u> $c \geq 1$.

$$\eta^{-1} = q^{-\frac{1}{24}} \sum_{n=0}^{\infty} p(n) q^n$$

$$U\eta^{-1} = e_p U \eta^{-1} \text{ "+" } \sum \alpha_i \phi_i .$$

$$p^*(p^n k) = \underbrace{\text{finite eigenforms}} + O(p^{nc})$$

<u>Lemma</u>: $c \geq 1$.

$\exists ?\, \phi \in M_0^{\dagger}(\Gamma_0(1))$

$U\phi = \lambda\phi \qquad 1 > v(\lambda) > 0.$

if ϕ exists

$\Rightarrow$ also exists a classical modular form

f of height weight

($\equiv 0 \bmod p-1$)

level 1.

~~stage~~

$v(a_p) = v(\lambda).$

Thm (Buzzard–Gee)

$$\Rightarrow \bar{\rho}_f\big|_{D_p} = \mathrm{Ind}_{\mathbb{Q}_{p^2}}^{\mathbb{Q}_p} \omega_2^{p-2}.$$
$$= \bar{\rho}_\phi\big|_{D_p}$$

Serre's Conjecture

$\omega^2 \otimes \bar{\rho}_\phi$ modular of level 7.
and weight $= 4$

$\Rightarrow$ contradiction

$\Rightarrow$ nothing of slope $\in (0,1)$.

FC5-9

$\phi \in M^{+}_{-\frac{1}{2}}(``\Gamma_0(1)")$. $p \neq 2,3$

$1 > v(x) > 0$.

$S(\phi) \in M^{+}_{-2}(\ )$

same slope

$\in M^{+}_{-2}(\Gamma_0(6)) \otimes \chi_{12}$

new at 2, 3

$U_2 = 2^{-2},\ U_3 = 3^{-2}$.

$\bar{f} \in S_6(\Gamma_0(6))^{2,3\text{ new}}$.

$U_2 = 2^{2},\ U_3 = -3^{2}$

FC5-10

The p-adic analysis of overconvergent eigenforms.

fix N, fix p.

$\{\phi_1, \phi_2, \phi_3, \phi_4, \ldots\}$.

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fix N.

Maass

$\{\phi_1, \phi_2, \phi_3 \ldots\}$

$N(T) = \{\#i \mid \lambda_i \text{ of } \phi_i \text{ is} < T\}$.

Weyl's Law

$$N(T) \sim \frac{\mathrm{Vol}(X)}{4\pi} \cdot T$$

$N_p(T) = \{ i \mid v(\lambda i) < T \}$.

Conj.
p-adic Weyl Law.

$$N_p(T) \sim \frac{\mathrm{Vol}(X_0(p))}{4\pi} \cdot T.$$

Known (Wan) correct upper bound

$\left(\begin{array}{l} p=2 \\ N=1 \\ k=0 \end{array} \right)$ correct lower bound up to a constant scalar

✓

FC 5-12

Overconvergent.
P-adic
Arithmetic
Quantum
Unique
Ergodicity.

ψ_i — distribution.

— zero set.