COURSE OUTLINE: IWASAWA THEORY OF ELLIPTIC CURVES ARIZONA WINTERS SCHOOL 2017

CHRISTOPHER SKINNER

1. Course Outline

Iwasawa theory was introduced around 1960 in the context of class groups of cyclotomic and other $\mathbb{Z}_p$-extensions of number fields. The 'main conjecture' of Iwasawa theory proposed a remarkable connection between the p-adic L-functions of Kubota and Leopoldt and these class groups, including among its consequences certain refined class number formulas for values of Dirichlet L-functions. This main conjecture was proved by Mazur and Wiles in the early 1980s.

Beginning with work of Mazur and Swinnerton-Dyer in the 1970s and especially in subsequent papers of Greenberg, the ideas of Iwasawa theory were extended to elliptic curves and – having been suitably recast in the language of Selmer groups – other p-adic Galois representations. Each instance has it's own 'main conjecture' (at least conjecturally!) relating certain Galois cohomology groups (the algebraic side) with p-adic L-functions (the analytic side).

The aim of this course will be to describe the Iwasawa theory of elliptic curves and – possibly – modular forms, state the associated main conjectures, and report on some of the progress that has been made on proving these conjectures and especially some of the arithmetic consequences. A rough outline of the course:

- Selmer groups of elliptic curves
- Main Conjectures
- recent results (and their proofs)
- arithmetic applications (to BSD, etc.)

2. Projects

Possible projects include:

1. *Missing cases of the main conjecture for elliptic curves.*

2. *Relating different main conjectures.*

3. *The structure of Selmer groups.*