Revised Course Outline
Grothendieck-Teichmüller Theory
Moduli Spaces and Multizeta Values

Arizona Winter School
March 12-16, 2005

Lecture I: We will start with the final part of the course notes, on linear Grothendieck-Teichmüller theory. Even though this part will make more sense, theoretically speaking, after having seen the big picture, beginning with it will allow the students to get started on their research project.

Lecture II: The following basic notions will be introduced: algebraic/arithmetic fundamental groups, moduli spaces of curves, their fundamental groups (the mapping class groups). We will have a look at the theory of dessins d'enfants.

Lecture III: We will start Grothendieck-Teichmüller theory, with the definition of the group $\widehat{GT}$ and a sketch of the proof of the basic theorem that the absolute Galois group $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ injects into $\widehat{GT}$. Then we will investigate Grothendieck's notion of the "Lego game" and the "two-level principle" for moduli spaces of curves, and state a concrete version of his "Lego Theorem". The idea here is that everything in moduli spaces and their fundamental groups comes from what happens in dimensions 1 and 2.

Lecture IV: We will introduce braided tensor categories and a higher genus (partial) analog. These are very formal and rather complicated notions from category theory, but they play a large role in all kinds of mathematics from motives to mathematical physics, so it is good to have seen them once. We use them to prove the Lego Theorem.

Research project: It is conjectured that all algebraic relations between multizeta values come from double shuffle relations. We will try to prove this for some of the known relations on multizeta values of small weight, some stemming from Euler's time and others coming from the Grothendieck-Teichmüller relations satisfied by the Drinfel'd associator.