Math 124 - Exam 3 Practice problems- Spring '06

1. A window frame has the shape of a rectangle with a semicircle on top. All four sides of the rectangle are part of the frame. The straight portions of the frame (the four sides of the rectrangle) cost 8$ per foot, and the curved portion (the semicircle) costs 12$ per foot. The total area of the window must be 20 square feet. Find the dimensions that minimize the cost of the frame.

2. $f(x) = xe^{-x^2}$
(a) Find all x values at which f has a local min or max.
(b) Find all inflection points.
(c) Find the global min and max over $0 \leq x \leq 1$.

3. Find the following limits exactly,

$$\lim_{x\to\infty} \frac{1-\cos(ax)}{x^2}, \qquad \lim_{x\to 0} \frac{1-\cos(ax)}{x^2}, \qquad \lim_{x\to\infty} \frac{\ln(x)}{\sinh(x)}$$

4. Let $f(x) = 5a^3x^2 - 2x^5$. Here a is a parameter; it does not depend on x.
(a) Find the critical points and determine if they are local min or maxs. Your answer should involve a.
(b) Find the global max over $x \geq 0$.

5. Consider the curve $x^3 + y^2 \cosh(y-1) = 2$
(a) Find the equation of the tangent line at the point $(1, 1)$ to the curve .
(b) Use your answer to (a) to find approximately the value of y so that $(1.01, y)$ is also on the curve.

6. A function $f(x)$ has the following properties:
(i) f has a local min at $x = 0$.
(ii) f has inflection points at $x = 2$ and $x = 5$.
(iii) f has a local max at $x = 3$
(iv) $f'(7) = 0$, but $x = 7$ is neither a local min or max.
On the interval $[0, 8]$, f has no other critical points or inflection points other than those given above. Sketch a possible graph of the **derivative** $f'(x)$.